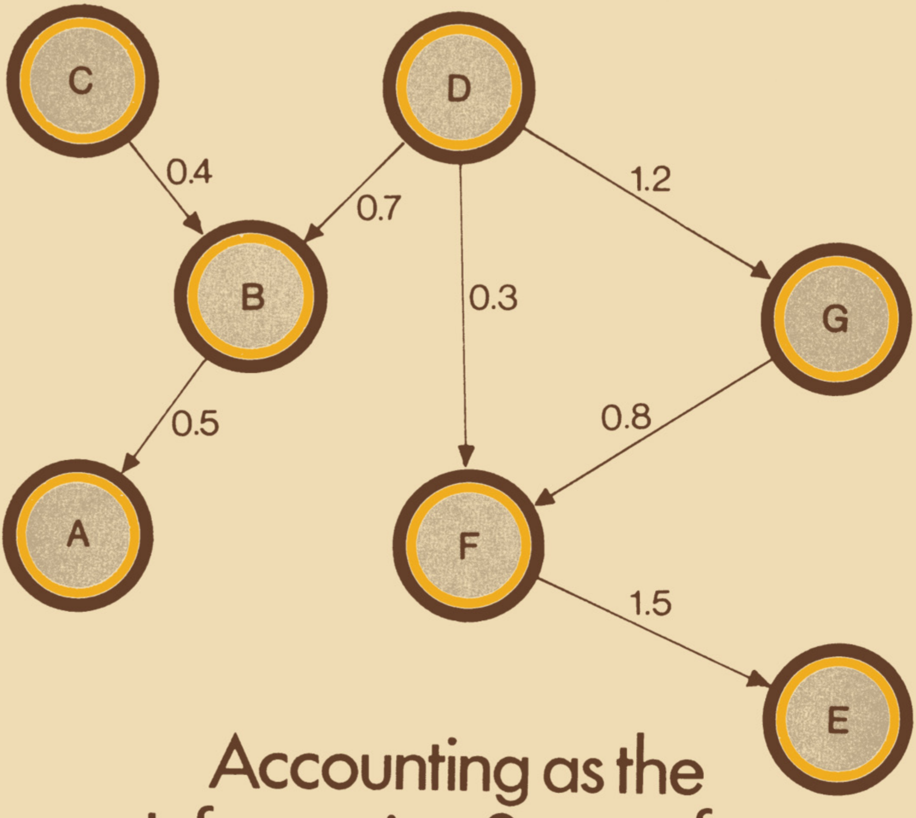
Trevor Gambling

MODERN ACCOUNTING



Accounting as the Information System for Technological Change

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By the same author

SOCIETAL ACCOUNTING

A ONE-YEAR ACCOUNTING COURSE

MODERN ACCOUNTING

Accounting as the Information System for Technological Change

TREVOR GAMBLING

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First published 1975 by
THE MACMILLAN PRESS LTD
London and Basingstoke
Associated companies in New York
Dublin Melbourne Johannesburg and Madras

SBN 333 16707 4

ISBN 978-1-349-02267-0
ISBN 978-1-349-02265-6 (eBook)
DOI 10.1007/978-1-349-02265-6

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Preface

This book is based, fairly loosely, on the author's unpublished doctoral thesis (42), presented at the University of Birmingham, England. For some years before and after its presentation the material formed the basis for a number of courses at post-graduate and final-year undergraduate levels, both for accounting specialists and some engineers. As a result of these field trials, the emphasis in this book is rather different from that in the thesis, or in the discussion papers which preceded it.

It should not cause surprise that a book on accounting should discuss material normally considered as a part of 'operational research'. The topics normally discussed as operational research are the application of scientific principles to decision making, and include model building, a variety of mathematical techniques and heuristic methods. Moreover although of itself separate from computer science and systems work, all this methodology has come into being because of the availability of advanced computational equipment. Accounting texts do not often reflect the influence of these developments; they tend either towards fairly abstract, philosophical treatments, or more usually to a detailed procedural approach which was designed originally for manual operation and fails to cover a variety of problems which are peculiar to large-scale enterprises. These problems arise from the complexity of the interrelationships between different parts of the enterprise, and the change that is taking place at various points within the system. Accounting must concern itself with large-scale mathematical model building if it is to be able to accommodate enterprises of this type.

The book, in short, sees accounting as an exercise in commercial systems engineering. The opening chapter tries to say something about the nature of 'change' as it affects the accounting system of an enterprise; it goes on to discuss how conventional accounting theory has sought to encompass change, and to suggest a theory of economic accounting which seems better adapted to cope with the problems of size, complexity and changeability. The next chapter emphasises that the accounting system is itself a model of the enterprise it serves, and contrasts various techniques of model building. One technique, called 'technological model building', is selected as being suited especially to providing a satisfactory model for this purpose, and its methodology is described in some detail.

The book next shows how the basic model is used as a 'cost model', primarily as a substitute for various methods of process-cost accounting. Thus far the work covered can be considered as an extension of the conventional material of advanced accounting. However the model is a comprehensive one, which can be adapted readily to a variety of programming techniques aimed at the ascertainment of optimal budgets. Here again the large size of system being modelled presents difficulties over the use of conventional programming methods, and it is necessary to consider how the basic model can be aggregated into more compact forms or 'decomposed' into a number of more manageable problems. This book does not cover mathematical programming itself; however it does describe a method of calculating an approximate solution to problems of this type using an heuristic method.

So far the book has considered the accounting system as a model of the physical processes of manufacture and distribution; the methodology contains a number of novel, or at least little-known features, but the concepts themselves are not new. Moreover the techniques have all been used in practice. The remainder of the book is more theoretical in its content. It seems possible to apply the large-scale modelling techniques developed in the first part of the work to the information system itself. Again, this idea is not new, but earlier writers who have considered it were hampered by having to use a less fully developed modelling methodology.

It seems possible to argue that one effect of the development of highly automated integrated data-processing systems is the elimination of a large amount of human intervention in the preparation of data, which could be described as an 'editing' function. At first sight, this editing function would seem to be no more than an accept/reject assessment of the reliability of the results obtained. However, it will be argued in Chapter 4 that the real nature of the function is rather more complex than this.

Essentially uncertain basic data are permitted to flow into the system and become part of increasingly complex hierarchies of information statements; it is important to recognise where the editing activity occurs in the system, and either reproduce it electronically or provide for human intervention in the process. This requires the construction of a model of the information system which shows clearly the ancestry of the various statements produced; since the information system itself is a model of the underlying technological reality, the model of the information system must also be capable of encompassing very large systems and of responding to change.

Apart from the location of editing operations, it is in any case useful to know the basic ancestry of the complex information statements, if only to be able to evaluate their reliability. The book closes with a discussion of possible methods of measuring the reliability of such statements, and the effect of such evaluations on the 'feedback' of cost variances into the information system.

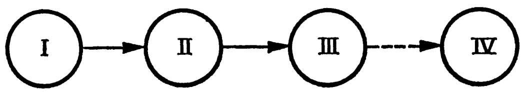
Some parts of the material have been the subject of papers in *Abacus*, *Accountancy*, *The Accounting Review*, *The Chartered Accountant in Australia*, and *Management Accounting* (New York); the papers are partially reproduced or closely paraphrased in the text.

1 Change and Accounting Theory

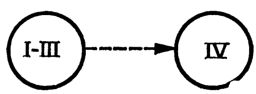
1.1 THE NATURE OF CHANGE

During the last few decades a good deal has been written about technological change and technological development. In describing its subject as 'the information system for technological change', this book is not very much concerned with the phenomenon of technological change in itself, nor even with its economic or financial implications. It does not deal, at one level of sophistication, with the problems of accelerated depreciation of fixed assets, nor, at another, with econometric models describing the relationship of expenditure on research to the development of new techniques. Instead, it considers how an accounting system, in its broadest sense, can adapt itself to conditions in which the underlying system (or 'real' system) which it is seeking to describe is subject to rapid and largely unforeseeable change.

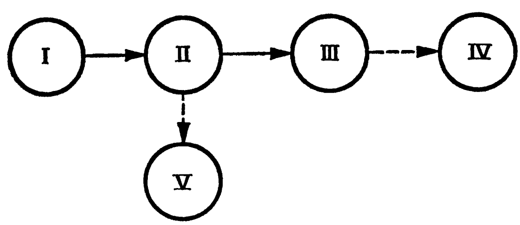
However before describing the changes needed in the basic technique of accounting in order to cope with such changes in technology, it will be useful to say something about the nature of innovation, so that the problems which it is likely to present to the accountant may be better understood. The assessment of these problems and the validity of the several techniques which might be used to resolve them depend upon the beliefs held of the probable directions of technological change in the particular circumstances of some enterprise. These in turn will depend upon the view taken of the processes by which creative thinking becomes possible. If innovation is only possible along predeterminable lines, it would be reasonable to prefer some concise and rigid accounting system which only permits change or growth at a few rather obvious points in the system. The real systems themselves are likely to comprise numerous manufacturing processes[1] with complex interrelationships, since otherwise change would present few accounting problems. An accounting system which allowed for changes at any part of so large a real system will add to its computational problems; a more rigid system would allow the aggregation or consolidation of large areas of the work.[2] A very simple example will illustrate this: suppose a product goes through a series of processes, numbered I through III, and a possible development might be the addition of a further process, IV. The complete systems would be illustrated (and mathematically modelled) as



However little of value would be lost by presenting the system, both visually and mathematically, as



On the other hand, this consolidation would be less desirable if it was necessary to consider yet another possible innovation, the development of the process V as a continuation of I and II only:



If the underlying system is part of an industry which is undergoing rapid technological development, it will be essential to select the accounting system which seems suited to the free 'bi-sociation' of ideas which Koestler (68) sees as the heart of every major innovation. Bi-sociation makes it possible to find improved methods and new concepts in any field of knowledge, by discovering new and unsuspected congruities and affinities between existing concepts. By its nature, the process must be unplanned – an intellectual jump onto a new plane of thought. Such jumps will not be the only source of valuable development, but if the further development of an existing idea is seen as producing decreasing returns to scale (as it usually does), the discovery of a new idea will place the developer in an area where higher returns become possible again. In the large, interrelated, multi-process enterprise, the result of any development will be the creation or discontinuance of relations between the processes, with or without the creation of new processes or the disappearance of existing ones. The problem for the accountant is to decide upon the degree of flexibility to be allowed in the information system for changes of this sort.

It can be seen that the essential difference between an accounting system which permits free bi-sociation between any of the elements of the real system and some less flexible version, is that the latter requires *a priori* assumptions to be made about the future operation, growth and development of the real system which is being portrayed. *A priori* assumptions are a necessary step in the process of bi-sociation itself, since the innovator first assumes that some affinity exists, and then sees if observable data support the idea; what must be avoided is a prejudgement of the existence or non-existence of supportable assumptions by the information system itself. Because they are themselves an essential process in the advancement of science or in the development of new products or processes, *a priori* assumptions should only be built into the accounting system where it is certain that they will not conflict with those required for the technological development of the underlying real system. The task of an information system is to reflect such developments rather than to mould them.[3]

1.2 CHANGE AND THE ECONOMIST

It has been stated that the economics of change are not part of the subject matter of this book. However a substantial body of work has been done by economists on 'production functions' (*vide* the well-known review article by A. A. Walters (111)), and it will be valuable to say something briefly about these, since the economist's experience in this field will give valuable insights into what forms of these 'functions' correspond most closely to the type of accounting system advocated in the previous section of this chapter. Essentially, a production function is an equation which gives the output of the system as the dependent variable governed by a variety of factors ('independent variables'). The equations are usually of a fairly abstracted nature and use concepts such as 'output' and 'labour' in a general sense, rather than considering the complexity of detail introduced by any specific manufacturing process. Thus a simple model will be

$$q = f(x_1, x_2, x_3 \ldots x_n),\ x_i > 0 \qquad (1.1)$$

where the output (q) is simply stated to bear some unspecified mathematical relationship (f) to some unspecified quantities of an unspecified number of inputs (x_i). For any given product of course, the relationship and the inputs can be specified, and the equation will still be true, but limited to that single case. Note that 'capital' and 'revenue' inputs are not distinguished here.

The mathematical relationship, or function, is usually taken to be continuous, which implies that an infinite number of possible combinations of these inputs is possible; 'the best utilisation of any particular input combination is a technical, not an economic problem.' (54 p. 44.) Moreover the equation is static, which is to say that its duration is such that

(i) no alterations are possible in the fixed inputs;
(ii) no changes are possible in the selection of technology, and
(iii) the time nevertheless suffices for the completion of the necessary production cycles.

The function is also 'homogeneous', to the extent that

$$f(tx_1, tx_2) = t^k f(x_1, x_2),\ t > 0, \text{ real};\ k \text{ constant} \qquad (1.2)$$

which is to say that if the inputs are increased by a factor t, output increases by a factor t^k. Where $k = 1$, the function is, of course, linear and this is the assumption underlying those accounting operations where costs are considered to contain 'fixed' and 'variable' elements.

It can be seen that this simple function is likely to be true in most circumstances but that it does not add greatly to one's appreciation of the economic problems of production. In fact more sophisticated equations have been developed for this purpose by the economists. However the very general nature of this particular function provides exactly the right approach for an accounting system, especially one which is required to reflect an underlying reality in circumstances where it is inappropriate to make *a priori* assumptions about changes over time. The fact that the function is static is seen to imply that no change at all is possible in the time during which the function is operative, but this does not mean that change cannot occur. Subject to the need to complete existing production cycles, the values of f and x_i can be changed at any time, and so bring the duration of the current production function to an end. The effect of this updating process is to substitute a new production function every time a change is made; no provision exists *inside* the function itself for making these changes. To the extent that it is possible to forecast what changes will occur over time, one could use a more sophisticated production function as the basis for the accounting system, but otherwise the model can only be updated by an external input of new technological data. The more sophisticated functions will often take account of technological change as a sort of disturbance in the system which explains the difference between the actual and the expected productivity of the operations.

1.3 THE STATIC ASSUMPTIONS OF TRADITIONAL ACCOUNTING THEORY

It is now possible to approach a little nearer to the main topic of the earlier part of this book, and show how a typical accounting system restates the simple production function just described. Every traditional bookkeeping system contains a written or unwritten 'chart of accounts' and 'manual of procedures'. The chart of accounts is a list of the nominal- and cost-ledger account headings, and thus sets out the way in which its transactions are to be allocated within the accounting system; the headings will reflect in greater or less detail the enterprise's physical processes of manufacture and distribution. If functional analysis is used with or in place of natural analysis, the chart will include a list of the production centres and overhead cost centres which comprise the real, underlying system.

The manual of procedures is the set of rules by which the books are written up. Obviously the original entries of the transactions with the outside world will be posted in accordance with their correspondence to account headings in the chart of accounts. However it is also necessary to have some rules for transferring some of the resulting balances between account headings, and these rules set out the relationships which are seen to exist between the various cost and production centres. As it stands, without the addition of any transactions, the chart-plus-manual can be seen to provide the unquantified x_i's, q's and linking f's of the basic production function. Nevertheless the traditional accounting system is likely to prove an unsatisfactory representation of the underlying reality for a number of reasons:

(i) It may reflect a number of previously accepted but now rather debatable views of accounting, such as full-absorption costing and the collection of costs into global cost-centres by arbitrary transfers of totals which do not clearly correspond to any relationship in the real system. For example it is a common practice to allocate, say office rents into administrative overhead, and then absorb the administrative overhead into production, although no physical connection is represented by the transfer.

(ii) The chart reflects only the technical processes and marketing strategies currently being used; this is acceptable for historical accounting but not for the preparation of budgets. There are no facilities in the chart for considering alternative products, methods of production or sales strategies which may be possible within the existing technology and environment.

(iii) Apart from what can be inferred from its headings, etc., the traditional accounting system does not contain any direct record about the technology and economic environment of the enterprise, although a good deal of technical and marketing information is usually retained at local level by the production and selling staff.

(iv) The chart of accounts and manual of procedures is seen as something to be imposed from outside the traditional accounting system. No machinery is provided for updating them automatically, so it is necessary to rely upon someone noticing and rectifying discrepancies between the accounting systems and the real system as they occur. When a system contains many thousands of processes (as defined in Note 1) mistakes are inevitable and very difficult to detect.

The purpose of this part of the book is to describe some techniques for including all available technological and marketing alternatives in the basic accounting system, and providing for their updating. It is not surprising that no facilities of this sort are available in the traditional system, since accounting was developed very much in its present form some 700 years ago, at a time when technological change was scarcely discernible, and in a society which would not have understood the concept of alternative marketing and production strategies. Moreover the industrial and commercial organisations of the period were not interrelated multi-process systems, so that any technological change which did occur, reflected itself as changes in the items bought and sold, or in the suppliers and customers themselves, and so did not require any internally originated action to reflect them in the books.

1.4 ECONOMIC ACCOUNTING

In general the creation of new headings in the chart of accounts will be the result of some major technological or environmental change; it will usually represent an extension of the real system of the enterprise. Extensions of this sort will almost always occur as a result of capital expenditure, or at least expenditure which could be capitalised. Here the capital expenditure will be the acquisition of plant, accommodation or other equipment for a new 'project' not hitherto undertaken by the enterprise. However one could not automatically link the creation of new account headings with capital expenditure, since traditional accounting theory also treats the replacement of plant, etc. on existing projects in the same way. The traditional treatment of so-called capital items can be criticised for other reasons: although capital items are separated out for profit ascertainment, their total cost is part of the cash flow in the period in which they are paid for. This technique also leads managers to see their capital budgets as something quite apart from their revenue budgets, although the two are in fact closely interrelated.

It seems reasonable to suggest that the traditional treatment of capital and revenue also proceeds from the type of business venture common at the time it was developed. Looms, kilns, mills and so on were bought at the outset of the enterprise; since their rate of technological obsolescence was very low, and manufacture was in any case labour-intensive, these basic items did not require replacement, as opposed to repair, during the life of the enterprise. As a result, it seemed reasonable enough to segregate this initial 'capital expenditure' from 'revenue expenditure', and then to calculate the 'profit' of the venture as the difference between the income which relates to some period and the revenue expenditure, less some allowance for the depreciation of the capital items.

This approach to profit is seen to be at variance with the economic production function which forms the basis for the accounting system, since that did not distinguish capital inputs. It is also at variance with a systems approach to accounting, which would be concerned to distinguish expenditure which changed the real system, from that which did not, rather than the separation of revenue and non-revenue items. If a completely logical defence is to be made of the systems approach, it is necessary to enquire into this matter of 'profit' in some detail:

1.4.1 *The objectives of business activity*

The objective of any business is to maximise its 'profits'. At least this is probably so from the point of view of the community at large, who want to see the largest 'added value', or difference between the value of the inputs into a business and the value of its output. A number of objections can be made to this concept; people commonly have non-financial objectives and it is necessary to weigh long-run and short-run profits. These can be countered, in part, by saying that the price-system *ought* to take proper account of *all* the social costs involved, and that the concern should really be to maximise the *present* value of all expected future profits. The latter argument can be justified by the existence of stock exchanges and other markets for business undertakings where it is possible to obtain present cash in return for the right to receive future income.[4]

1.4.2 *The nature of 'profit'*

The basic policy of the business is therefore not specifically concerned with the profit of any one time period, but with those of all foreseeable time periods. On the other hand, if we assume that a right to receive a given income in the future has a different (and usually smaller) value than the right to receive the same amount immediately, we are concerned with the distribution of future profits over time. In short, it is necessary to be able to define very precisely what is meant by the profit of a period. In general profits are taken to be the maximum amount which could be distributed by way of dividend without reducing the basic capital value of the business and so we see that the concept of profit is dependent on the value we place on the basic capital, and also upon what we conceive that basic capital to be. If this basic capital is supposed to bear some relation to the present value of the undertaking, and the present value depends on future profits, it is apparent that the terms 'profit' and 'income' have to be carefully defined if a circular argument is to be avoided.

1.4.3 *The conventional concept of accounting profit*

The approach to these problems in conventional accounting is to locate 'income' in time, usually with reference to the creation of a legal liability to pay for the goods and services; 'expenditure' is then matched against this income as far as is possible, or otherwise identified with some period of time in the same way as the income. Following from this, 'capital' items are distinguished from 'revenue' items; the former being the expenditure which cannot be identified with the current trading period and consequently needs to be carried forward against future periods. A number of difficulties follow from this approach:

(a) The convention of recognising income and expenditure on the creation of a legal liability divorces this concept of periodic profit from the underlying flow of cash within the business. It is apparent that the important aspect of any transaction is not the creation of the liability, but its eventual discharge, if any; meaningful financial decisions must be taken in terms of cash flows, since it is only in these terms that solvency can be judged in the end.

(b) The division between capital and revenue items poses problems with regard to the maintenance of the basic capital. Usually these are seen in terms of difficulties over currency depreciation, so that the amounts retained by charging historical costs against profits are not sufficient to replace the items for further trading. In fact the problem is the essential uncertainty of the future in every respect of which currency fluctuation is only one example. The decision to carry forward any item as capital implies a confidence that future income will arise against which it can properly be charged; the traditional technique carries the proviso that expenditure on capital items which cannot be recovered in this way should be written off against current profits as 'obsolescence'. Since this approach to the profits is concerned with costs rather than values, the question of maintaining capital intact can only be dealt with by the creation of external reserves, which require some subjective evaluation of how much of the future capital expenditure of the business should be financed without further injections of finance, *or* without the use of internal funds which shareholders might otherwise have expected to be paid out as dividends.

1.4.4 *Alternative views of profit*

It has been shown that true periodic profit depends upon the value that can be placed upon the undertaking at the beginning and end of the period in question. This value depends in turn upon the expectations of future income, and it is apparent that the future income to be used here is by no means confined to that income which arises from the existing plant, inventory, debtors, patents or labour force. Neither is it possible to relate any part of that income to particular pieces of the present or future undertaking; all one can do is to estimate the marginal income of some part, namely the *additional* income arising because that part of the undertaking is available, which is a very different thing.

This idea is recognised in conventional accounting as the asset 'goodwill'. Although the convention rarely permits this item to appear as much more than a nominal amount, it is clearly seen to be a measure of the excess value of the whole undertaking over the historical value of the physically present net assets. The fact that goodwill is almost never included at any scientifically calculated value suggests that the profits arrived at by conventional means are mis-stated to some extent. In particular, these profits will be at variance with the concept of return on capital commonly used in the evaluation of capital expenditure decisions, where all values are in terms of future income streams rather than historical costs.

On the other hand the financial records of an enterprise do not contain estimates of future income, but only of the historical cost of currently extant assets and liabilities. In the absence of generally accepted techniques for estimating future activity, such estimates might seem likely to be either non-existent or entirely subjective surmises. As a result, accounts are prepared on the basis of what Palle Hansen (52) calls 'historiography'; the value of the enterprise is taken instead as some function of the historical cost of its present assets and liabilities. In a world where technology and environment (including the value of the currency) are stable, this approach is reasonable, since the price paid must presumably measure the lower bound of the value set by the purchaser on the contribution to future income expected from the asset at the time of acquisition. Without any change in those expectations, cost-based values must retain validity over the life of the asset. What is missing in such a case is only the present value of the future net income which will arise from assets not yet acquired.

This argument is less convincing where substantial technological and environmental changes are occurring, unless perhaps one could believe that the prices paid took account of probable changes of that type. Even where the latter belief was justified, it is obvious that change is likely to make the proportion of the true present value relating to *unacquired* assets into a much more important item.

Both the airiest of economic theory and the earthiest of business practice agree that true 'income' is the net cash flow within the period, adjusted for the change that has taken place over the period in the present value of *all* future net cash flows.[5] Moonitz and Jordan have shown that under conditions of complete certainty the income is the equivalent of the interest used to discount the future cash flows over the life of the enterprise; clearly the actual cash flows precisely cancel out the estimates originally made of them. Where the future flows are completely uncertain, the income is the unadjusted cash flow only.

A worked example, based on the 'Model of complete certainty' from Chapter 5 of Moonitz and Jordan's text (81):

(1) Statement of cash flows and their discounted values at 31 December 19x1

		Project 1
		£
Basic investment		19,374
Add Return of 6% p.a. for 19x1		1,162
		20,536
less Net cash flow	£	
Cash receipts during 19x1	10,000	
Cash payments during 19x1	6,000	4,000
Valuation of the enterprise at 31 Dec 19x1		16,536
Add Return of 6% for 19x2		991
		17,527
less Estimated net cash flow for 19x2		5,000
Valuation of the enterprise at 31 Dec 19x2		12,527
Add Return of 6% p.a. for 19x3		753
		13,280
less Estimated net cash flow for 19x3		6,000
Valuation of the enterprise at 31 Dec 19x3		7,280
Add Return of 6% p.a. for 19x4		436
		7,716
less Estimated net cash flow for 19x4		3,000
Valuation of the enterprise at 31 Dec 19x4		4,716
Add Return of 6% p.a. for 19x5		284
	£	5,000
less Estimated net cash flow for 19x5	2,000	
Notional value of terminal assets	3,000	5,000
		— —

The effect is to treat the series of cash flows as if they were annuity payments of a varying amount; if these net flows are withdrawn from the business the capital would be exhausted, but the question of reinvestment is a separate matter from the ascertainment of true profit.

Palle Hansen considers the case where the estimates of future income have to be revised from time to time and thus give rise to 'capital' adjustments in the periods when the revisions are made. These revisions must be made because changes in environment or technology either have occurred or have become apparent for the first time; discrepancies between the current cash flows and their earlier estimates may not necessitate adjustments of this type, since they can arise from uncertainty in the data and need different treatment (see Chapters 6 and 7). In short, the value of an enterprise is the present value of its future expected cash flows and not at all a function of the historical cost of its presently-held assets. Practice bears this out, since the price which was paid for the stock of a defunct company has no effect on the payment, if any, eventually received from the liquidation.

Whenever a subsequent change is made in the forecast of the cash flows, it is necessary to recalculate the returns and valuations back to the original investment. If one thinks of the business as a single investment project, such an adjustment would have to go back to the foundation of the firm, since no distinction is made between capital and revenue expenditure in this type of accounting and subsequent replacements of plant are simply taken up into the cash flows. On the other hand, where it is possible to distinguish a number of separate projects[6] within the enterprise, the adjustments to cash flows would only affect the relevant project and so only need adjustment back to its inception. In any case, technological change is more likely to reflect itself in the development of new projects and the abandonment of the existing ones. Each project would have its own rate of return in these circumstances, and it might be necessary to arrive at some average rate to discount cash flows which cannot be attributed to any one project.

Continuing the example, it might be useful to consider the effect of a change in technology requiring the introduction of a new project in year 19x3 and the phasing out of project 1 in 19x4 (see page 14).

The changes in the estimated net cash flows of Project 1 necessitate the recalculation of the rate of return back to 19x1

(1) Statement of cash flows and their discounted values at 31 December 19x3

	Project 1		*Project 2*		*Total*
	£	£	£	£	£
Value brought forward		12,527			12,527
less Adjustment of valuation, due to re-estimation		367			367
Adjusted value at 31 Dec 19x2		12,160			12,160
Basic investment		—		17,953	17,953
Add Return for 19x3	(5%)	608	(10%)	1,795	2,403
		12,768		19,748	32,516
less Net cash flow					
Cash receipts during 19x3	10,000		7,000		
Cash payments during 19x3	5,000	5,000	3,000	4,000	9,000
Valuation of enterprise at 31 Dec 19x3		7,768		15,748	23,516
Add Return for 19x4	(5%)	388	(10%)	1,575	1,963
		8,156		17,323	25,479
less Estimated net cash flow for 19x4	4,000			6,000	
Estimated value of terminal assets	4,156	8,156		—	14,156
Value of enterprise at 31 Dec 19x4		— —		11,323	11,323
Add Return for 19x5			(10%)	1,132	1,132
				12,455	12,455
less Estimated net cash flow for 19x5				7,000	7,000
Valuation of enterprise at 31 Dec 19x5				5,455	5,455
Add Return for 19x6			(10%)	545	545
				6,000	6,000
less Estimated net cash flow for 19x6			4,000		
Notional value of terminal assets			2,000	6,000	6,000
				— —	— —

and the new rate is found to be 5 per cent p.a. The adjusted value is therefore calculated as:

	£
Original investment	19,374
Add return of 5% for 19x1	969
	20,343
less Cash flow	4,000
	16,343
Add return of 5% for 19x2	817
	17,160
less Cash flow	5,000
	£12,160

1.4.5 *Accounting in 'economic' form*

This then is the theory of accounting under technological and environmental change. It is suggested that such changes make the traditional historiographic position untenable and necessitate the valuation of an enterprise solely in terms of its estimates of the future. The records to be kept might not be very different for such a system; the traditional accounts would be needed for accounts receivable and payable and for the cash for control purposes. It would suffice to maintain only physical records for fixed assets and inventories, but since analysis of the receipts and payments would be needed for forecasting purposes it is probable that normal accounts would be kept for these items also. However all the problem areas of traditional accounting theory could be ignored; depreciation, obsolescence, accruals and prepayments, LIFO and FIFO are of no concern, since a 'balance sheet' would consist of a certified 'present value' of the enterprise. Similarly, the revenue account would be reduced to a certified 'internal rate of return', while interim accounts would comprise the grounds for believing that the cash flows were still on course for the current forecasts. 'The availability of profits for dividend purposes' would also disappear as a problem, because all 'profits' will be calculated in terms of the net cash flows expected to become available. *Prima facie* the whole of the net increase in the cash balance can be distributed in dividends, subject to any decision of the directors to retain all or part of it against future cash requirements which otherwise would have to be met from borrowings or fresh issues of stock. As in traditional accounting however the subscribed capital can be retained by limiting dividends to the amount of the calculated 'profits'. Inflation might be taken into account as a particular cause of the capital adjustments just described.

1.4.6 *Some practical problems*

(a) In theory the 'internal rate of return' could be either fixed or floating. A fixed rate would be that minimum expected rate of return which serves as a cut-off criterion for investment proposals; a floating rate would be whatever rate was necessary to discount the income stream from a project down to its original investment amount. It is well known that the latter technique presents a mathematical difficulty, because under some circumstances more than one such rate can be found, but on balance the floating rate would seem more suitable for accounting purposes, as opposed to those of investment decision making. This is because there is little virtue in considering an enterprise's income as that of an hypothetical marginal firm, and then calculating its goodwill as the excess of the value of its income over the actual cost of its investment.

(b) The form of the 'economic accounts' might be:

- (i) a statement of the cash flows and their value at the beginning of each year of the estimates;
- (ii) details of the current cash flow, and its distribution and retention.

Examples of the first statement have already been given. The relevant current cash flow statements would be as shown on pages 17 and 18.

(c) The estimates can only cover a finite number of years, unless one thought it proper to assume some estimate of perpetual income at the conclusion of the most distant plan. It would be necessary to assume a liquidation otherwise.

(d) Strict application of theory would require one to take account of profits expected from projects which have not yet been commenced, but which are envisaged, and one might wonder what would be the effect of this proposal on the burden of taxation. Not only does the technique bring forward profits, but it also brings forward obsolescences and increased costs of replacement. Hansen's view was that historiographic accounting tended to overstate profits, especially for a declining firm. If one believed that technological change made every firm into a declining firm, to the extent that every one of its projects has a built-in element of obsolescence, there could be long-term tax advantages to be drawn from such a change in the basis of accounting theory.

(2) Statement of cash flows for the year ended 31 December 19x1

	£	£
Proceeds of stock issue		(25,000)
less Costs of issue	2,626	
Investment in Project 1	19,374	22,000
		(3,000)
Cash from receivables		(10,000)
less Payments		
Raw material	2,000	
Labour	3,000	
Other items	1,000	6,000
		(7,000)
less Proposed dividend		1,000
Retention for 19x1		(6,000)
Retention in previous periods		—
Total cash retained		£(6,000)
Represented by:		£
Balance of cash at 31 Dec 19x1		2,800
Investments at cost (M.V. £5,300)		4,200
		7,000
less Provision for proposed dividend		(1,000)
		£6,000

(2) Statement of cash flows for the year ended 31 December 19x3

	Project 1		*Project 2*		*Total*
	£	£	£	£	£
Investment in Project 2		—		17,953	17,953
Cash from receivables		(10,000)		(7,000)	(17,000)
less Payments					
Raw materials	1,500		1,000		
Labour	2,500		1,000		
Other items	1,000	5,000	1,000	3,000	8,000
		(5,000)		13,953	8,953
less Proposed dividend for 19x3					—
Negative retention for 19x3					8,953
Retention in previous periods					(10,000)
Total Cash retained					£(1,047)
Represented by:					£
Balance of cash at 31 Dec 19x3					(1,047)
Investment at cost					—
					(1,047)
less Provision for proposed dividend					—
					£(1,047)

1.5 A SYSTEMS APPROACH TO ACCOUNTING

Economic accounting provides the theory for a systems approach to accounting. It remains to summarise what the theory requires in the accounting system itself:

(a) Profit and value are functions of forecasts of the future rather than of historical cost. Since the concern is with the future rather than the past, it is natural that the accounting system will comprise the existing processes, and all known alternative processes which exist or may arise in the forecast period.[7] Of course, only the processes which it is currently proposed to undertake will be used in the calculations, but it will be seen in later chapters that non-acceptable alternatives need to be borne in mind in a changing, interrelated system, since apparently unconnected changes may make them more desirable in future.

(b) It follows that the type of accounting to be used here is in effect standard costing rather than historical process-costing. In any case, this would be preferable for the large interrelated systems under consideration. Horngren says: 'A standard cost system not only eliminates the intricacies of weighted-average versus *FIFO* inventory methods; it also erases the need for burdensome computations of costs per equivalent unit. The standard cost *is* the cost per equivalent unit.' (62 p. 477.) Using standard costing, each process becomes a cluster of cost centres whose efficiency can be judged by comparing (i) its actual consumption of intermediate products from within the system with the standard usages and (ii) its actual conversion costs with the standard expenditure for those items. It is apparent that the historical process-costing techniques could only be used for a quite small number of interrelated processes in practice; real systems commonly contain 10,000 or more such processes.

(c) The forecasts will be forecasts of periodic cash flows, rather than attempts to allocate 'income' and 'expenditure' to the future periods. The most obvious effect will be a need to take account of the time value of money in arriving at standard costs and (especially) at standard contributions. A product whose cash cost is £100 per unit and whose production cycle over a number of processes takes six months is obviously more expensive than one which also costs £100 but has a cycle of only three weeks. Similarly a cash sale of £200 is more valuable than a credit sale for £200 to a customer who takes two months to pay – and might never pay at all! This is important for calculating the optimal strategy, since items which take a long time to make or are sold to dubious customers are commonly sold at an apparently high price to take account of this. If the contributions obtainable are not discounted, but taken at their gross value, what is really expensive and risky business will always seem the most attractive alternative open to the enterprise.

(d) The variances disclosed by the comparison of the historical results with the budget are either controllable, when hopefully they will be corrected in future, or uncontrollable when they indicate a need to revise the standard costs (see Chapter 7). This revision of standards is another type of technological change, which does not require any extension or reduction of the structure of the real system. The forecasts are continually recalculated using the revised costs, to produce the capital adjustments described above.

(e) The total 'structure' of the real system is seen to comprise all possible processes open to the enterprise, whether they are in use or not; hence the equipment and working capital needed for any one of them may or may not exist. A process is incorporated into the forecast of a project through being selected by the budgeting operation, which will allocate a production budget to it; a project is abandoned when no allocation is made to it, although its component processes will not necessarily disappear from the structure. At any one time this total structure will contain an 'operating structure' consisting of the currently operating processes and the projects appearing in the accounts will be those necessary for this operating structure. If the production budget calls for the employment of a project which does not yet exist, it is necessary to budget also for the finance of the initial investment needed.[8]

(f) Traditional accounting theory claims to maintain an automatic inventory of an enterprise's assets; unless both systems continue to run in parallel it is necessary to consider how the economic system might compare in this respect. Although the latter system concerns itself with cash flows, records would be maintained for cash, debtors, creditors and stores inventories, as part of the physical control system. A physical inventory of the plant and equipment would also be needed for the same reason, and also to provide the data on the capacity of the processes for the budgeting operation. These inventories would not form part of a balancing system of accounts, as has been seen; however it is suggested that the real security given by a double-entry system of final accounts is somewhat overrated in view of the ease with which adjustments can be made and subsequently justified on capital accounts. Physical inventories and an adequate system internal check provide most of the safety that is available under either system.

A forecast might seem to be more subjective in nature than an historical cost. This is true enough in the case of an isolated forecast, but when the enterprise prepares and updates its forecasts regularly, it is possible to investigate the reasons for any changes which have occurred in the currently held expectation of the future. Moreover it is possible to assess the reliability of earlier forecasts against the actual performance. It will be seen in the later chapters of this book that accounting data frequently have a substantial subjective element which affects the degree of confidence felt for them and can be assessed in such terms (see Chapter 6).

1.6 NOTES

1. A 'manufacturing process' needs to be carefully defined throughout this book; it is always an operation or series of operations which have a single homogeneous product or output (in the case of joint production, a single homogeneous group of products). This means that in most of these discussions, 'process' and 'product' are interchangeable terms, since the output of an intermediate *process* is seen as an intermediate *product*, and not as a partly finished final product.

2. Some degree of aggregation is an essential part of the accounting process: any form of categorisation must involve this technique. In general the literature of accounting research has considered the effect of aggregation on 'success' in decision making, e.g. Barefield (5), Butterworth (15), although Lev (72) is more concerned with the data base than the user.

3. The pursuit of some general principles for the construction of management information systems has occupied a good deal of space in the journals over the years. The basic dilemma is whether it is possible to retain a supply of data for decision making purposes in advance of knowing what decisions will be taken. Moravec (82) and Ackoff (1) present the general discussion, while Hindman and Kettering (55) give a very practical illustration. Marshall (77), and Feltham and Demski (37) adopt a decision-theoretic approach which some may find helpful. It is the more general management scientists, such as Argyris (3) and Mitroff (79), who consider the essentially limiting nature of any information technique which interposes an 'expert' between a user and the whole body of his data. The latter writer commends the Hegelian approach – 'one only stands to understand an issue, any issue, when he has witnessed the strongest possible debate take place about that issue'. The issue will be discussed further in Chapter 6, since it also affects the probability and reliability of the aggregated data: see also Note 1 of Chapter 4.

4. The question of whether 'social net product' can differ from 'private net product' has been a matter of dispute since Pigou raised the issue at the turn of the century. The topic is discussed at length by the author in another place (47); perhaps the briefest summary would be to say that the machinery which links any one enterprise with society as a whole is too complex to be reflected adequately in the accounts of the enterprise itself. That work also discusses the influence of culture on accounting theory. Readers should note that the discounting technique advocated here is essentially a western capitalist's approach: other cultures may be much less

interested in future benefits. Because the enterprise is a sub-system in a much larger system we call 'society', we must face the fact that it is not easy to attach an unquestionable measurement to its functions!

5. Many accountants and accounting theoreticians will object to the proposed methods because they involve making forecasts about the distribution of cash flows over future time which cannot be provided with a reasonable degree of accuracy. On the other hand, the firm's capital investment programme is supposed to be justified in just this fashion. Although Seed (99) casts very reasonable doubt on the good faith of many of the figures produced for the latter purpose, it is the author's view that a logically justifiable accounting system must be both *future-based* and *project (i.e. systems)-based*: he feels that Hansen (op. cit.) deals with the practical difficulties of the approach in a quite satisfactory fashion.

6. A new project is created whenever a substantial extension is proposed in the real system; it will be a matter of judgement whether an extension is seen as a new project or as a variation of an existing one. Usually, but not essentially, a project will be sufficiently expensive to need some special arrangements to be made for its initial finance.

Every process is part of one of the projects of the enterprise; a project usually involves an investment in one or more pieces of plant or other equipment which provide the capacity to accommodate the processes.

7. An interesting paper by Elmaghraby (34) discusses the construction of representations of systems ('networks' or 'graphs' – see Chapter 2) which avoid the need to construct networks for each possible turn of events. He attempts to do this by attributing logical 'switching' functions to some of the nodes (see Chapter 2) in the graph. Of course, what is being advocated here is not the construction of separate networks for each alternative, but rather that of a network which shows them all. It will be seen in subsequent chapters that this approach assumes the additivity of direct, variable costs, whichever path is in fact followed. This is a rather strong assumption; some of the difficulties are discussed in a recent paper by Ijiri and Itami (65).

8. An enterprise's budgeting operations falls into three parts: (i) the long-term budget, whose length is determined by the lead time necessary to install the equipment needed for new projects; (ii) the medium-term budget, whose length is that period within which no additional plant can be brought into operation; and (iii) the short-term programme or production schedule itself. The 'structure' reflected by the information system will comprise data for all three budgets, the total structure being the basis for (i) and the operating structure for (ii). In addition, there will be what might be called a 'current structure' forming the basis for (iii), consisting of the plant which is a serviceable condition, the current orders and so on.

2 Model Building and Planning

2.1 ISOMORPHISM AND HOMOMORPHISM

The previous chapter has referred from time to time to the need for an accounting system to 'reflect' the underlying real system. It is now necessary to consider more precisely how this process of reflection is to be carried out. The literature on accounting theory (e.g. Chambers, 18) occasionally uses the rather formidable term 'isomorphism', by which is meant the degree to which the accounting statements contain items which correspond clearly to the visible physical features of the real system. The technique of economic accounting could be and has been (Chambers, 19) attacked because the final accounts do not contain identifiable entities called 'inventory', 'debtors', 'plant and equipment' and so on. Obviously the technique does not do this, but reasons have already been adduced for claiming that such identification may not give a correct picture of the overall economic position of the enterprise.

Accordingly it seems probable that no quantified summaries can provide the sort of one-for-one correspondence looked for as isomorphism. What is usually more satisfactory is some quantified 'model' which will behave in the same way as the real system when used in particular decision making exercises without necessarily reflecting its physical appearance; if a term was necessary, perhaps 'homomorphism' might be appropriate here. The term 'model' is itself frequently encountered in discussions in this area and implies some concept which is a reflection of the underlying reality. Any sort of accounting system is therefore a vehicle for a model and something has to be said about the type of model which would be contained in an accounting system having the degree of flexibility and other features described in the previous chapter. It can be deduced from this that more than one model might be constructed for a given real system; a reasoned choice amongst these will involve discussion of the basic techniques of modelling.

2.2 THE 'MODEL' AND THE SYSTEM

A model is simply an idea which one might have of the reason things work in the way they seem to do![1] A model is not essentially a *mathematical* model; strictly, the set of simultaneous equations which is commonly described as 'a model' is merely a quantification based upon the model itself. By its nature, a model is a proposal rather than a statement of fact, and so it does not have to be unique. If the observed phenomenon is that the Sun always rises in the East and sets in the West, two possible models would be 'because the Earth is moving around the Sun' or, 'because the Sun is moving around the Earth.' It should be noted that both supply an entirely satisfactory explanation of the observed phenomenon, at least in the unquantified form in which it is stated above. It is only when the models are used to explain more complex observations, such as why the Sun rises and sets at different hours at various times in the year, or why it rises to different heights in different parts of the world at the same time of the year, that one model might seem preferable to the other. It is also apparent that if the justification for a model has to be proven from measured observations, or the model is to be used to forecast future events in measurable terms, it is necessary that the model be stated in a mathematical form.

A rather foolish example will illustrate the technique; it has not been chosen out of flippancy, but only to prevent the preconceptions which a more sensible industrial example would imply. Someone has invited a mixed party of men and women to dinner; how much of which type of sherry should he buy for them to drink when they arrive? Obviously, he could get in a dozen sweet and a dozen dry, in the belief that this represented far more of either wine than was likely to be demanded. This is a common real-life solution of such problems in industry as well as in party giving; the cost of holding a surplus stock might be less than the embarrassment of running out! Again, he might ask friends who had held similar affairs how much their guests seemed to get through. This too is a recognisable industrial solution; one's own or other people's experience could be used to give an *ex cathedra* answer to a problem.

However suppose that the party was to be a fairly large one, so that the would be host had no confidence that even a dozen bottles of each would be safe against a stock-out; or that his own or his friend's experience had been with much smaller events, or all-male groups. A more sophisticated approach than the earlier solutions is now needed; if he knew, or thought he knew how his guests' thirst for sherry was regulated in the first place, he could work out some sort of forecast of what this particular group would consume. To do this a model is needed; he might merely say, 'The more people I invite, the more drink they get through.' On the other hand he could add more details to this idea, which (like the original idea) may or may not be true in fact; he could add, 'but the men usually drink more than the women,' and 'most women prefer sweet sherry,' and so on. This unquantified model could be restated more formally in this fashion: 'the consumption of sweet sherry at a party is related to the number of men present, plus the number of women present; the same can be said for the consumption of dry sherry.'

Essentially a model claims that certain end results, or 'dependent variables', are caused by the behaviour of certain other factors in the situation, which are known as 'the independent variables'. Here the dependent variables are the required quantities of sweet sherry and the required quantities of dry sherry; the independent variables are the number and sex of the guests. Using mathematical terms, the dependent variables are 'functions' of the independent variables, so the model could be written as

$$\begin{aligned} C_s &= f_s\,(M + F) \\ C_d &= f_d\,(M + F) \end{aligned} \qquad (2.1)$$

This is sometimes called the 'structural equation', and the significance of the notation used is obvious enough. These equations not only set out the variables which are related by the function, but also the mathematical sign linking them; here it is addition. The quantification of this structural model could be arrived at in at least three ways:

(a) The host could rely on his own or other people's observations of guests' drinking habits: this might be, 'On average, 75 per cent of women select sweet sherry and the remainder dry, while whatever they selected they consumed one-tenth of a bottle; men chose their drinks 80 per cent dry to 20 per cent sweet, and consumed one-eighth of a bottle.'

(b) Alternatively, rather than observing people actually drinking he might attempt to quantify the problem along these lines: 'I saw ten men and twelve women attend a party; during the party two bottles of sweet and three bottles of dry sherry were consumed'; and probably go on to list the input of guests and the output of empty bottles at a number of other parties as well.

(c) Another alternative (which might be less far-fetched when applied to an industrial process than to sherry parties!) would be to realise that sherry consumption is not governed directly by the sex of the drinker, but by the relative body weight, fluid content and so on, which themselves are governed by his or her sex. Accordingly the model could conceivably be quantified in terms based on some assumptions about male and female biochemistry.

The choice between these three methods is very important and will depend on a variety of factors. For the moment, only (a) will be considered. The quantification of the structural model is then straightforward, especially since the assumption has been made in this observation that the function is linear, which is to say that if the number of guests and their consumption of liquor were to be plotted on a graph, the plots would form a straight line; this means that the independent variables only need to be multiplied by a constant figure (known as a parameter):

$$C_s = (0{\cdot}2 \times 0{\cdot}125 =)\ 0{\cdot}025\ M + (0{\cdot}75 \times 0{\cdot}1 =)\ 0{\cdot}075\ F$$
$$C_d = (0{\cdot}8 \times 0{\cdot}125 =)\ 0{\cdot}1\ \ M + (0{\cdot}25 \times 0{\cdot}1 =)\ 0{\cdot}025\ F \quad (2.2)$$

Thus a party of thirty men and twenty-six women can be expected to consume 2·7 bottles of sweet sherry and 3·65 bottles of dry sherry.

Two further points are illustrated by this example. Firstly, even if the identification and quantification of the independent variables were quite correct, it would still be possible to obtain a variety of models, depending upon the degree of aggregation or disaggregation introduced. So far the independent variables have been taken as the numbers of male and female guests; an equally 'correct' model could be built by selecting 'the number of guests' as the sole independent variable, and saying

$$\begin{aligned} C_s &= 0{\cdot}05\ G \\ \text{and } C_d &= 0{\cdot}0625\ G \end{aligned} \tag{2.3}$$

The parameters in these equations are just the arithmetic averages of the consumptions used in (2.2); they are therefore the figures which would have been obtained by observing the overall consumption of sherry at parties attended by equal numbers of men and women. This formula would give forecast consumptions of 2·8 sweet and 3·5 dry for the mixed party of fifty-six people under consideration, because it does not allow for the fact that the sexes are to be distributed in different proportions in the 'forecast period' from what they were in the 'observation period'. Similarly a finer, less aggregated analysis would be possible by further sub-dividing the two groups by age, occupation and so on.

Secondly, whatever degree of aggregation and method of quantification are selected, past experience or observations are being used to forecast the future. It may be that what happened in the past is only a very poor guide to the future. Past experience of drinking with barmaids will not be of great use in planning for a party of Sisters of Charity. This dilemma is central to most of the discussion in the early chapters of the book; provided that conditions remain unchanged, simple, highly aggregated models will be quite adequate for forecasting future events. Moreover it is not necessary that the observations should have been very accurate, since repeated use under the same conditions permits a feedback from the ensuing over- or under-estimates to amend the structure and parameters of the model. But where major technological or environmental change occurs it is necessary to have a very precise structural model with a low degree of aggregation, so that the effect of change can be assessed therein.

2.3 THE TECHNOLOGICAL MODEL

The three techniques of quantification outlined in Section 2.2 have precise counterparts in industrial model building practice.

Technique (a) required the parameters to be calculated by detailed observation of the physical processes, from which 'average' or 'expected' consumptions could be estimated. Turning to an industrial example, it is apparent that a standard costing system may be expected to provide information of this type, and the previous chapter has suggested that the need to base profits and valuations on the future rather than on the past will require the accounting system to be maintained on that principle in any case.[2] However even a moderately sized enterprise will require a very large accounting system to reflect its real system, if the real system is extended to include the costs of all its technological alternatives.

The technique to be described has been developed as a practical exercise largely in the chemical manufacturing industry in Germany and Great Britain, although there is no special feature which inhibits its use in other types of manufacture. Certain branches of chemical manufacture involve large-scale multi-process batch-production, so that comparatively straightforward pieces of chemical plant very commonly do have a variety of technological alternatives open to them, from which they are expected to select the most profitable. Since it is also necessary to take account of marketing restrictions in making these plans, the production programme for a single piece of plant will usually be made up of many more than one 'process', as defined in the previous chapter. Using this definition, a single division of such a company might have to take account of 4000 (Wenke, 112), 10,000 or perhaps 80,000 distinguishable intermediate and finished products; moreover the industry is one in which both technological development and changing costs are to be expected.

In addition chemical intermediates can often be mixed together in a variety of ways, and so form different finished products through a long chain of intermediate processes. In the past this combination of numerous stages of manufacture, intricate interrelationships and rapid change has posed a serious bookkeeping problem. Even where a standard costing system was combined with a high degree of automation, it became hard to establish the costs of the final products by carrying forward the standard costs of previous intermediates ('ancestors'), and adding in the conversion costs of the current process, to arrive at a further standard cost to carry forward to later processes ('descendants'). This was because one could not be sure that the recalculated standard costs had been passed on to all the *possible* descendants of any intermediate, in the absence of some formal machinery for updating the chart of accounts and manual of procedures. Any such machinery would itself need to be automated, since any sort of visual presentation of 4000 or more interrelated processes is bound to be so large as to be totally confusing.

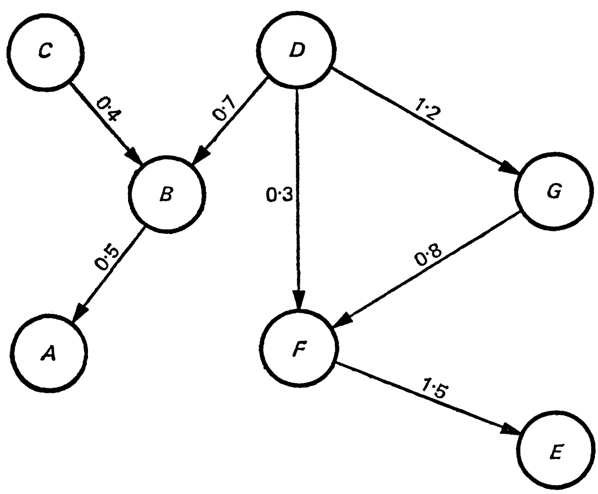
The solution has been to build what is called a cost-model, whose base is a 'technological matrix' containing the standard inputs necessary to produce a single unit of the output (or yield) of each process. Before describing this technique in some detail, it should be observed that input-output matrices of a very similar appearance were developed some years ago for macro-economic (or National Income) purposes by W. W. Leontief (71); a number of papers (16, 35, 36, 64, 76, 93) have speculated on the possibility of extending his technique to micro-economic (industrial planning) problems.[3] There are some difficulties over doing this, since Leontief's method of constructing his matrices is completely different from those to be set out here.[4]

2.4 THE GOZINTOGRAPH AND ITS MATRICES

As has been explained, practical examples of the system being considered are much too large for visual display. However an exposition of the technique will be assisted by considering an unnaturally small example containing only seven interrelated processes. Their relationships can be illustrated initially by the following diagram, which is sometimes called a 'gozintograph'. It is advisable to use this rather light-hearted title for flow charts of this type, since then there is no doubt as to the significance of the chart and the figures which appear on it (see Figure 2.1).

It remains to restate this graph as a matrix; this transformation requires, strictly, acquaintance with the mathematical disciplines of graph theory and matrix algebra, although the actual technique to be employed will be entirely obvious if a reader does not allow himself to worry over the mathematical significance of what is occurring![5] For the moment it is only necessary to know that the circles on a graph are known as 'nodes' (or 'vertices') and that the arrows are known as 'arcs'. The arrowheads on this graph indicate the direction of the flow of production along these arcs and the nodes represent the processes (and their single, homogeneous outputs). The coefficient on the arcs represents the number of units of the ancestor needed as the standard input to make one unit of the user product.[6]

FIGURE 2.1 A 'gozintograph'



Similarly matrix algebra is simply a technique whereby a number of pieces of information can be arranged relative to their positions on two (or occasionally more) scales of values, rather akin to co-ordinate geometry; thus

		Scale 'B'			
		1	2	3	4
	1	*x*	*x*	*x*	*x*
Scale 'A'	2	0	*x*	*x*	*x*
	3	*x*	*x*	*x*	*x*
	4	*x*	*x*	*x*	*x*

where the individual pieces of data can be described as x_{11}, x_{12}, . . . x_{ij} . . . x_{mn}, using a convention that in the subscript numerals the first item (i) designates the row and the second (j) the column. Obviously this 'matrix' (or 'array') is a convenient method of storing and retrieving very substantial amounts of data in a compact form, but it also has the facility of being a 'complex number' (in the sense that $\frac{7}{8}$ is a more sophisticated concept than 78). This means that matrices are capable of being added, subtracted, multiplied and divided, under appropriate circumstances, as if they were ordinary numerals; the convention is to describe the matrix of x_{ij}'s as $[X]$, so that $[Z] = [X + Y]$ can be written to represent the matrix summation where $z_{ij} = x_{ij} + y_{ij}$, and so on.[7] The basic function of this algebra is the solution of sets of simultaneous equations, as will be seen.

Returning to the gozintograph (Figure 2.1) the relationship which is being studied is that existing between the processes, or the input-output relationship. This means that a node-to-node matrix is required, rather than the several varieties of arc-to-node or arc-to-arc matrices which could be based upon this graph. In this matrix, the columns will represent the output nodes, while the rows indicate the necessary input nodes (see Figure 2.2).

FIGURE 2.2

		Outputs						
		A	B	C	D	E	F	G
	A							
	B	0·5						
	C		0·4					
Inputs	D		0·7				0·3	1·2
	E							
	F					1·5		
	G						0·8	

The significance of this matrix is obvious; for example, 'To make one unit of B requires inputs of 0·4 unit of C and 0·7 unit of D'; 'To make one unit of E takes 1·5 units of F,' and so on. Following Vazsonyi's (110) notation, this will be referred to as the *next assembly* matrix, and designated [$\mathcal{N}$]; n_{ij} is seen to be the standard usage of the ith intermediate product by the jth, which would be found in a traditional standard-costing system. [$\mathcal{N}$] is the basic 'technological matrix' of the system.

[$\mathcal{N}$] however has two deficiencies; it takes no account of the conversion costs of the process, as opposed to the material transferred from elsewhere in the system, nor does it show the indirect usages of intermediates through a chain of processes. A process will almost always have two sources of input, which are either 'endogenous inputs' in the form of intermediates from other processes in the same system, or 'exogenous inputs', in the form of raw materials and conversion costs from the world outside the system.[8] As seen by [$\mathcal{N}$], each output is a dependent variable of the inputs listed in its column; however the inputs themselves are dependent variables of their own inputs, while the truly independent variables are the exogenous inputs which are not shown in this matrix at all. These problems can be resolved as follows.

If g_i represents the gross output of the ith process for both internal consumption by other processes and external sales, c_{ij} is the consumption by the jth process of some part of g_i, and if y_i is the net output of the ith product which is available for sale, it is possible to write

$$g_i - \sum_{j}^{m} c_{ij} = y_i \qquad (2.4)$$

namely that the net output of the ith process is the difference between the gross output and the sum of all the internal consumptions of the ith product elsewhere in the system. Now if the further assumption is made that this internal consumption is represented by a linear function, so that a doubling of the input will produce twice the output, it is possible to say also

$$c_{ij} = n_{ij}g \qquad (2.5)$$

or that the jth process's consumption of the ith intermediate represents some constant fraction of the gross output of the jth product; this n_{ij} is of course the appropriate item from [$\mathcal{N}$]

above. Substituting Equation (2.5) in Equation (2.4), the latter becomes

$$g_i - \sum_j^m n_{ij} g_j = y_i \qquad (2.6)$$

This equation could be written for each process in the system, given a list of gross outputs and the appropriate technological input-output coefficients. The system shown in Figure 2.1 would calculate the net output (y_i) for all m processes, as the right-hand side of the following seven simultaneous equations, given g_i:

Product (i)	g_i (given)	$-\sum_j^m - n_{ij} g_j$ (given)		$= y_i$ (calculated)
A	4,000			= 4,000
B	7,000	− 0·5 × 4,000	(A)	= 5,000
C	6,800	− 0·4 × 7,000	(B)	= 4,000
D	19,910	− {0·7 × 7,000 0·3 × 3,500 1·2 × 5,800}	(B) (F) (G)	= 7,000
E	1,000			= 1,000
F	3,500	− 1·5 × 1,000	(E)	= 2,000
G	5,800	− 0·8 × 3,500	(F)	= 3,000

However it has been pointed out that matrix algebra can be used to solve sets of simultaneous equations; since the real problem will be very large and matrix operations are well-suited to processing by computer, practical problems will always be handled in this way. The columns g_i and y_i are themselves 7 × 1 matrices; they would usually be called 'column vectors'. Using matrix notation, the Equation (2.6) can be written

$$\begin{aligned} [G] - [N]\,[G] &= [Y] \\ \text{or } [I - N]\,[G] &= [Y] \end{aligned} \qquad (2.7)$$

where $[I]$ signifies an identity matrix, which is the matrix algebra equivalent of the numeral 1; this will be illustrated in due course.

Usually it is some desired $[Y]$ which is given in the problem, rather than the [G], as in Equation (2.7), so that the dependent variable in the equation will be [G], which must be restated in terms of the given independent variables, [N] and [Y]:

$$[G] = \frac{[Y]}{[I-N]} \tag{2.8}$$

which is conventionally written as

$$[G] = [I-N]^{-1}\,[Y] \tag{2.9}$$

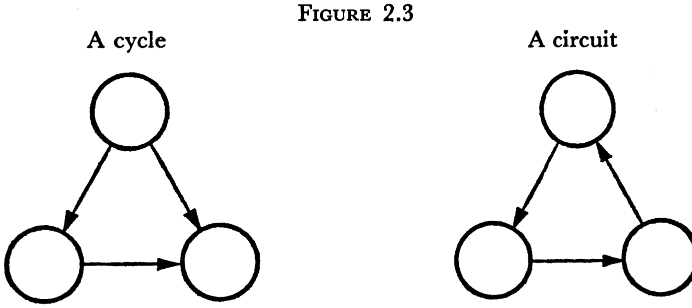
The matrix $[I-N]^{-1}$ is called 'the inverse of $[I-N]$'; 'the inverse of [X]' is the matrix-algebra equivalent of $1/x$ (or the reciprocal) in normal arithmetic. It can be seen that in this case the matrix forms as it were a ready reckoner showing the gross output need from the whole system to produce one unit of net output from each process. This matrix is the same as that arrived at by Leontief (op. cit.) for macro-economic work, although the coefficients are calculated on different principles; however Leontief calls the matrix [N] matrix [A] and so his inverse is $[I-A]^{-1}$ also. This notation is not followed here, because as has already been mentioned, the pioneering work in this field by Vazsonyi (op. cit.) has been followed instead. In fact Vazsonyi would call $[I-N]^{-1}$ the matrix [T], for 'total assembly', and this title will be preferred here too, because it enables the individual coefficients in the inverted matrix to be referred to directly as 't_{ij}'s'.

2.5 THE INVERSION OF VERY LARGE MATRICES

It is important to bear in mind at all times that the [N] and [T] of any real system will be matrices of the dimension of 4000^2 or more, which cannot really be thought of as capable of existence outside a fairly substantial computer. Even so, it is unlikely that the computer will be able to effect, say, the inversion of so large a matrix using the normal Gaussian methods, or determinants. However, the usual form of the type of matrix which is based upon a flow chart of some kind will often permit the use of more economical methods. A graph formation whose properties are extensively discussed in classical graph theory is the 'tree', which is defined as 'a finite connected graph with no cycles and possessing at least two vertices; except for the pendant vertices, every vertex of a tree is an articulation point.' It might be noted in passing that the theory is prepared to contemplate infinite graphs, and graphs consisting of several sub-graphs which are not connected, but the remainder of this jargon needs to be interpreted carefully, since the concepts will underlie much of what follows.

(a) A cycle is a finite chain which begins and ends with the same vertex x; the arcs which make up a chain need not all point the same way, and where the arcs in a chain of this sort *are* uni-directional, so that all the nodes are 'strongly connected', the formation is described as a 'circuit'. A circuit is always a cycle, but not vice versa (see Figure 2.3).

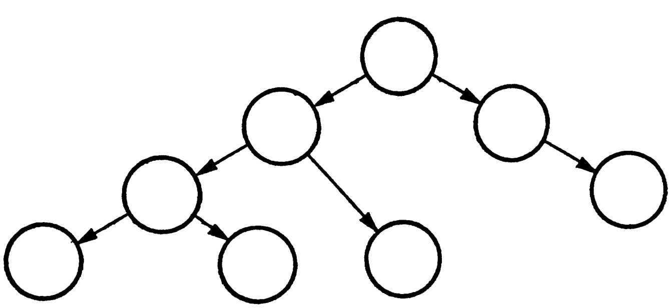
FIGURE 2.3



(b) A vertex is an articulation point if its deletion from the graph would result in the creation of a sub-graph which is no longer connected; a vertex is pendant if it has only one arc incident to it – namely a first, or more usually last item.

The appearance of the tree is distinctive (see Figure 2.4).

FIGURE 2.4 A tree



A refining process might well have this form, as might a straightforward machine assembly line, only with the arrows reversed. A chemical process can sometimes, but not often, involve both cycles, when products with a common ancestor run together, and also circuits, usually when some chemical is 'recovered' as a waste product in a process and recycled back to the basic process which uses it. In fact the existence of a cycle is of no particular significance; for the present purpose, the peculiar advantage of a tree-with-cycles in a production model is that it is also uni-directional (trees do not always have to be uni-directional, of course). This means that the process/products can be arranged in ancestor/descendent relationships, and then manipulated sequentially, so that when any one product comes to be dealt with, all its inputs will have already been evaluated, or inverted. This can be seen if the matrix $[I-N]$ for Figure 2.1 is restated as its underlying simultaneous equations:

$$
\begin{array}{lllllll}
a & & & & & & = y_a \\
-0{\cdot}5a & +\; b & & & & & = y_b \\
 & -0{\cdot}4b & +\; c & & & & = y_c \\
 & -0{\cdot}7b & & +\; d & & -0{\cdot}3f \;-\; 1{\cdot}2g & = y_d \\
 & & & & e & & = y_e \\
 & & & & -1{\cdot}5e & +\; f & = y_f \\
 & & & & & -0{\cdot}8f \;+\; g & = y_g
\end{array}
$$

Obviously a and e have no internal users, while b and f have only a and e. Similarly c and g contain only b and f as internal consumptions, while d has b, as well as f and g. This establishes the order of the flow of production through the system, in reverse. If the 'generation' of a product is designated γ, one could write:

γ_1 comprises Product D
γ_2 comprises Product C and Product G
γ_3 comprises Product B and Product F
while γ_4 comprises Product A and Product E

If one referred back to Figure 2.1, it will be seen that this order is not unique – indeed in the accepted sense it is not even correct, since Product C has no ancestors and could be fairly described as γ_1! The reason for this apparent contradiction is that Product C is given γ_2 because it is the immediate ancestor of Product B which is in γ_3; if the matrix $[I - N]$ is rewritten in γ-order, it will fall into a visibly triangular form,[9] and it should be noted that this would still happen if C *were* put into γ_1. It is the relative order of *directly related* products which is significant here, rather than the relative order of products which are only related collaterally (see Figure 2.5).

FIGURE 2.5

	D	G	C	B	F	A	E
D	1	– 1·2		– 0·7	–0·3		
G		1			–0·8		
C			1	–0·4			
B				1		– 0·5	
F					1		–1·5
A						1	
E							1

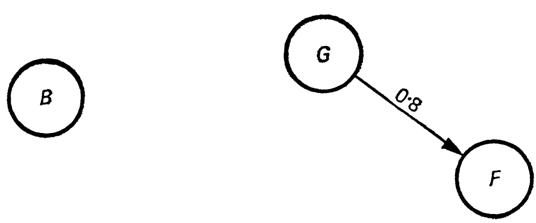
Before describing the method by which this matrix can be inverted with greater economy of space, it is necessary to recall that the practical examples of such matrices are very large and very complex. Moreover they will be presented to the model builder initially in some order which may be either random, or ranged on some other principle than their relative generations. The nodes in Figure 2.1 were lettered so as not to fall into generation order; Figure 2.2 is ranged in alphabetical order and does not therefore exhibit a visibly triangular form. Since the gozintograph cannot be drawn, it is necessary to devise an algorithm which will be capable of taking the 'recipes' or bills-of-material as they come and allocating them to appropriate generations unseen. An algorithm is a mathematical device which operates sequentially, so that it works away on a problem until it achieves some desired state and then stops, automatically. One such algorithm is described by Wenke (112); it is not the only algorithm which might be used, but it has the advantage, for illustrative purposes only, of being based on a search through $[\mathcal{N}]$ itself.[10] In fact Wenke's algorithm falls into two parts, the first of which is aimed at simple tree-and-cycle formations, while the second resolves the dimensions of circuit formations. The first part involves locating the nodes which form the very top and very bottom items on the graph. Returning to Figure 2.2, it will be seen that A and E have blank rows (because they have no descendants), while C and D have blank columns (because they have no ancestors). This indicates that A and E can be allocated to γ_n, since the total number of generations is not yet known, while C and D become γ_1. Note that this algorithm has produced a different order to that used above, although both orders will give a triangular form, as was explained.

If *both* the rows *and* the columns relating to the products already located are removed from the matrix, to produce what Wenke calls his 'reduced relationship matrix',[11] it is possible to carry out the same operation again:

	B	F	G
B	0	0	0
F	0	0	0
G	0	0·8	0

This shows what is the position if the original graph in Figure 2.1 was actually 'topped and tailed' to leave this disconnected graph (see Figure 2.6).

FIGURE 2.6



B and G now have blank columns and so are allocated to γ_2, while F goes to γ_{n-1}, because of its blank row. B also has a blank row and it is obvious why this is so from the reduced graph; here too it can be seen that it is a matter of indifference whether B is rated γ_2 or γ_{n-1}. The matrix has now been reduced completely, and γ_{n-1} and γ_n can be redesignated as γ_3 and γ_4. The important thing about this algorithm is that it could be programmed to sort out a much larger graph of this type just as easily, and without needing any visual display.

Ignoring for the moment the possibility of a circuit being present in the graph, the following algorithm will compute $[I-N]^{-1}$ or $[T]$, once the matrix has been rearranged or indexed into its proper order:

$$t_{ij} = \begin{cases} 0, & (i > j) \\ 1, & (i = j) \quad i, j = 1, 2, 3 \ldots m \\ \sum_{k}^{j-1} t_{ik} \cdot n_{kj} & (i < j) \end{cases} \tag{2.10}$$

Note that this equation produces $[T]$ (or $[I-N]^{-1}$) from $[N]$ only.

The last alternative states that every coefficient (n_{kj}) in the as yet uninverted column for the *j*th output is used to multiply every coefficient (t_{ik}) in the already inverted row relating to the *i*th input; the (mathematical) products for each input are then summed to form the appropriate coefficient in the *j*th inverted column. This operation is made possible by the triangular form of the matrix, which ensures that the rows and columns relating to every input of the *j*th product have already been inverted.[12] In the same way, it is possible to write the t_{ij}'s over the top of the n_{ij}'s in the computer array, since once a column has been inverted it is not necessary to refer back to the uninverted form; this effects a considerable saving in space.

The $[T]$-matrix for Figure 2.1 is shown on page 40.

It will be recollected that Equation (2.6) was used to calculate a net output(y_i) from a given gross output (g_i); it was pointed out that the problem in reality was usually to calculate the necessary gross output for the production of some given net output, as shown in Equation (2.9). If each of the coefficients in

		Outputs					
	D	G	C	B	F	A	E
D	1	1·2	0	0·7	0·3 0·96 }	0·35	1·89
G		1	0	0	0·8	0	1·2
C			1	0·4	0	0·2	0
Inputs B				1	0	0·5	0
F					1	0	1·5
A						1	0
E							1

the columns of this [*T*]-matrix are multiplied through by the corresponding y_i of the earlier example, and the rows are then summed (which is the effect of the matrix operations in Equation (2.9)), the result is the original g_1 vector:

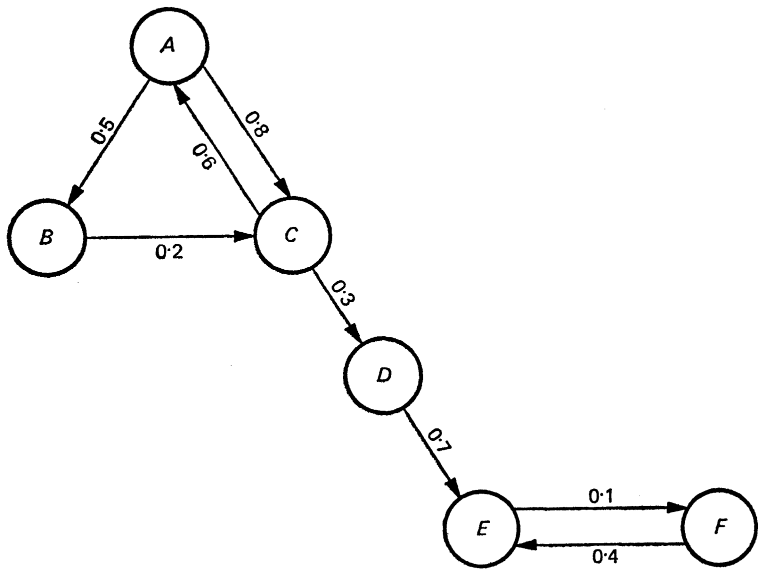
$$[T]\,[Y] = [G]$$

	D	G	C	B	F	A	E		g_i
D	7,000	3,600	0	3,500	2,520	1,400	1,890	=	19,910
G		3,000	0	0	1,600	0	1,200	=	5,800
C			4,000	2,000	0	800	0	=	6,800
B				5,000	0	2,000	0	=	7,000
F					2,000	0	1,500	=	3,500
A						4,000	0	=	4,000
E							1,000	=	1,000
(y_i)	(7,000)	(3,000)	(4,000)	(5,000)	(2,000)	(4,000)	(1,000)		

2.6 CIRCUIT FORMATIONS

Where circuits occur in the underlying graph, neither will this algorithm produce the correct inversion, nor will the 'topping and tailing' algorithm allocate the products to the appropriate γ-order. The γ-order is of itself less important here, because all items in a circuit must belong to the same generation, since they are all each other's descendants *and* ancestors, and in any case they cannot be placed in a triangular form. However a real system will not often consist of a single large circuit, but rather of a number of quite small ones, interspersed amongst normal 'tree'-formation networks, and it is necessary to identify the circuits and the products which comprise them, so that they may be removed and so expose the top or tail of the graph once more. This process is illustrated by Figure 2.7.

FIGURE 2.7 A graph with circuits



It can be seen that this graph covers just three generations, A, B and C in γ_1, D in γ_2, and E and F in γ_3. However this fact is not apparent from the matrix $[\mathcal{N}]$, and if that part of Wenke's algorithm which has been described previously were to be applied here, it would be unable to reduce the matrix, since there are no blank vectors. $[\mathcal{N}]$ is shown on page 42.

The fact that the equivalent relationship matrix cannot be reduced further in this form indicates that one or more circuits are present in the system. Wenke refers to circuits as 'cyclical nets', and his algorithm in fact locates what he calls 'maximal cyclical nets' only. A circuit (or cyclical net) is a sub-graph (H) of *m* vertices contained in the graph (G), wherein each vertex is strongly connected with the others, as has been stated previously. This circuit or cyclical net is said to be 'maximal' only if every other circuit in G is either itself a sub-graph of sub-graph H or contains no vertices at all in common with sub-graph H. In Figure 2.7 therefore there are three cyclical nets: (ABC), and (AB) and (EF), but only (ABC) and (EF) are maximal.

FIGURE 2.8

		Outputs					
		A	B	C	D	E	F
Inputs	A		0·5	0·8			
	B			0·2			
	C	0·6			0·3		
	D					0·7	
	E						0·1
	F					0·4	

In any circuit, every product is its own ancestor over *m* generations, where *m* is the number of products in the circuit. Wenke's method is to test the reduced matrix initially to see whether such a relationship exists over only two generations; if this does not reveal a circuit, he then tests sequentially for three-, four- or more generation relationships. This test involves the construction of what are called 'the second (third, fourth, . . . *n*th) power of the reduced relation matrices'. The reduced relation matrix is of course the sub-matrix of $[R]$ (see Note 11) which was used in the earlier illustration of the algorithm; if this sub-matrix is called $[R]^{(1)}$ the power matrices based upon it can be written $[R]^{(2)}$, $[R]^{(3)}$, . . . $[R]^{(n)}$. It was stated that $[R]$ and hence $[R]^{(1)}$ were matrices showing the node-to-node relationships, $[R]^{(2)}$ shows the node-to-node-once-removed relationships of the graph, and so on. It can be seen that $[R]^{(2)}$ can be obtained by comparing each item in a row with every item in the corresponding column; where both vectors show a value greater than zero for the same product, there is a two-generation relationship between the row and the column. This process is generalised with the aid of Boolean algebra as the equation

$$r_{ij}^{(n)} = \sum_{k}^{j} r_{ik} \cdot r_{kj}^{(n-1)} \text{ (Boolean)} \qquad (2.11)$$

Again, this is a version of the basic formula for matrix multiplication and represents a Boolean exponentiation of $[R]$ to the power of n. Boolean algebra is a tool of mathematical logic which is well suited to computer work; it is used here because it will select maximum and minimum values. In Boolean arithmetic,

'+Boolean' ≡ 'the logical *or*' $\therefore$ $x+y$ (Boolean) ≡ max. (x, y)
'×Boolean' ≡ 'the logical *and*' $\therefore$ $x \times y$ (Boolean) ≡ min. (x, y).

Using these rules, $[R]^{(2)}$ for Figure 2.7 becomes

FIGURE 2.9

	A	B	C	D	E	F
A	1	0	1	1	0	0
B	1	0	0	1	0	0
C	0	1	1	0	1	0
D	0	0	0	0	0	1
E	0	0	0	0	1	0
F	0	0	0	0	0	1

It will be seen that this matrix does supply the necessary information; for example in row C, it is apparent that C is related to B (through A), to itself (through A) and to E (through D) over two generations in each case. Circuits exist wherever a 1 is to be found on the diagonal of the matrix, since this means that the item is its own ancestor; however these circuits are not necessarily the maximal cyclical nets which are preventing the further reduction of the matrix. It is necessary to refer back to $[N]$ (or $[R]^{(1)}$) to establish which nodes comprise any one net.

For the purposes of establishing the γ-order any circuit can be treated as a single node and the internal coefficients can be ignored, which means that $[R]^{(1)}$ could be notionally rewritten as Figure 2.10.

FIGURE 2.10

	A/C	B	D	E/F
A/C	(–)	1	1	0
B	1	0	0	0
D	0	0	0	1
E/F	0	0	0	(–)

Returning to the original $[\mathcal{N}]$, the 'topping and tailing' operation will now be able to allocate the maximal cyclical net (EF) to γ_n, and then the node D to $\gamma_{(n-1)}$: the reduced relationship matrix is now

$[R]^{(1)}$

	A	B	C
A	0	1	1
B	0	0	1
C	1	0	0

$[R]^{(2)}$ can be reduced similarly to

$[R]^{(2)}$

	A	B	C
A	1	0	1
B	1	0	0
C	0	1	1

This has located a circuit between A and C, but it can be seen that this is not maximal because the 'notional' $[R]^{(1)}$ becomes

	A/C	B
A/C	0	1
B	1	0

which means that the A/C cannot be removed as yet. $[R]^{(2)}$ must therefore be multiplied by $[R]^{(1}$ to produce the next power matrix as

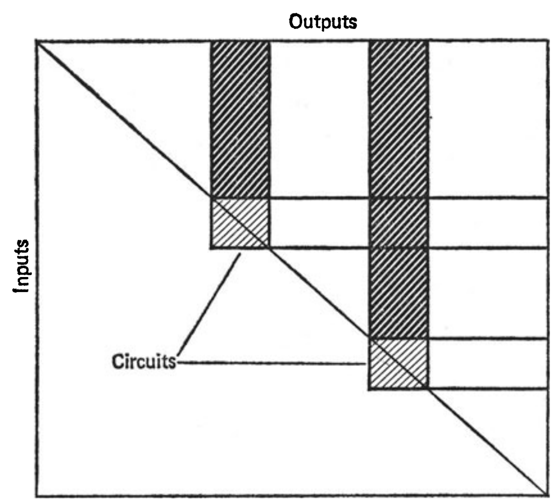
$[R]^{(3)}$

	A	B	C
A	1	1	1
B	0	1	1
C	1	0	1

which shows that a circuit exists over A, B and C, which in turn are seen to have no ancestors (or descendants), and can be allocated to γ_1 to clear the relationship matrix.

The $[\mathcal{N}]$-matrix given in Figure 2.8 is therefore correctly set out in γ-order, but does not have and cannot have a triangular form. It is not really possible to invert such a matrix sequentially, although an algorithm was devised to approximate to this in my thesis (42). However these circuits will occur as comparatively small irregularities in large but predominantly triangular matrices (see Figure 2.11).

FIGURE 2.11 A schematic $[\mathcal{N}]$-matrix



The circuits show themselves as projections below the diagonal; *provided that the normal 'tree' algorithm is used first to enter the full ancestry of all the products used by the circuit into the heavily shaded areas on Figure 2.11*, the sub-matrices formed by both could be inverted by normal methods. An algorithm has been devised to at least approximate the inverted version of the heavily shaded area (the usages by the circuit of its ancestors)[13] and the sequential algorithm could then continue until the next circuit is encountered.

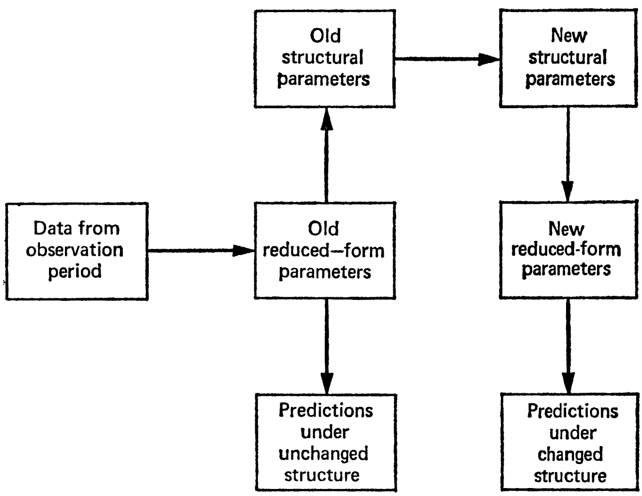
2.7 THE ECONOMETRIC MODEL

Returning to the remaining methods of quantification outlined in paragraph 2.2, method (b) involved consideration of the overall inputs into a system and outputs from the system, without directly taking account of the technical processes which occur within the system. There are considerable advantages to be derived from such a technique, if it can be used successfully, principally because the real systems are very large, while the intermediate processes have no significance apart from their effect on the finally saleable products.[14] As a result, in practice large areas of the technological matrix become effectually redundant, as will be demonstrated in the next chapter. The second technique is in any case the normal one used for building macro-economic models, where the number of the underlying processes hardly permits of a more detailed treatment; this is usually dealt with as part of the science of econometrics.

One aspect of econometrics is concerned with a rigorous study of postulates and theorems in mathematical economics and mathematical statistics, but another deals with empirical studies of real economic behaviour. In general these latter studies are in the field of macro-economics rather than micro-economics, but the technique is capable of constructing models of the type being discussed here. A basic assumption of econometric work is 'that parameters and random disturbances are unobservable' (Christ (21) p. 248); this means that the real system is always to be viewed as a 'black box' with some observed pattern of behaviour. From this the econometrician proceeds first to construct a maintained hypothesis, or more correctly a series of such hypotheses, which identify certain variables as affecting the behaviour of the system and establish the relationships between them. These hypotheses are in fact unquantified 'models' at a fairly high level of aggregation.

As was mentioned in Section 2.2, a high level of aggregation is acceptable where no change occurs between the period when the model was observed and the period to be forecast by it.

FIGURE 2.12 The sequence of inferences for making predictions under changed and unchanged structure



Ideally the maintained hypothesis will be the base of a series of structural equations, which will describe the actual working of the system in adequate detail; clearly in the case considered here of a micro-economic industrial system, these structural equations will be an equivalent of the technological matrix $[\mathcal{N}]$, although usually in less detail. From these equations it is necessary to extract a 'reduced form' of the equations, each of which will contain only one dependent, endogenous variable expressed in terms of the exogenous independent variables. The reduced form equations are an equivalent of the matrix $[\mathcal{T}]$, to the extent that they will map the basic inputs of the system into their final saleable outputs.

However if there is no expectation of change in the structure (and hence no need to change the variables or parameters of the structural equations) it is not necessary to construct the hypotheses formally in this way, and the reduced form or final parameters can be inferred directly from the observations. This means that the econometric model can construct its equivalent of [*T*], without considering [*N*] at all. Provided the variables are correctly identified[15] the reduced form can be an apparently irrational confection, and in the case where no change is expected there is no need to rationalise it; if it works for the observation period it will work for the forecast period. This is not the case when the structure can change, when the forecast can only be effected if the original structure and the nature of the change can be defined. The procedure is illustrated by Figure 2.12, from Christ's book (op. cit.), shown on page 47.

Here is how such a model might be built for the system shown in Figure 2.1. The behaviour of the system over say eight periods is summarised in the following time series:

(A)

Final net outputs
(in thousands of units)

	Products						
Period	A	B	C	D	E	F	G
1	4,000	5,000	4,000	7,000	1,000	2,000	3,000
2	1,000	3,000	5,000	4,000	2,000	6,000	2,000
3	3,000	4,000	3,000	6,000	1,000	3,000	2,000
4	4,000	5,000	4,000	5,000	2,000	2,000	4,000
5	2,000	4,000	2,000	8,000	3,000	2,000	2,000
6	3,000	5,000	5,000	5,000	3,000	3,000	3,000
7	1,000	6,000	2,000	1,000	1,000	1,000	8,000
8	2,000	8,000	1,000	8,000	2,000	2,000	2,000

Assume that the inputs into the system are two raw materials, X and Y, and a single class of labour; the costs[16] of the system for the same period have been analysed as

(B)

Period	Inputs in £'000 Raw Material X £'000	Raw Material Y £'000	Labour £'000
1	34,000	59,730	151,530
2	32,000	60,570	150,670
3	26,000	53,760	132,460
4	34,000	63,120	167,720
5	32,000	60,270	148,870
6	38,000	67,800	178,500
7	23,000	54,900	147,100
8	23,000	69,000	165,200

(C) A possible maintained hypothesis[17] might be

A = f (X + Y + L), where L signifies labour,
B = f (X + Y + L),
C = f (X + L),
D = f (Y + L),
E = f (Y + L),
G = f (Y + L);

this hypothesis is itself in a reduced form, so one might attempt to find the necessary parameters by using a normal multiple-regression program. In the event this produced the following matrix of parameters only after the additional variable 'O' was admitted:

	'O'	X	Y	L
A	- 697·683	+ 0·108	- 0·016	+ 0·006
B	- 1,429·966	- 0·248	+ 0·022	+ 0·081
C	- 1,243·496	+ 0·235	—	- 0·017
D	- 2,873·599	—	+ 0·731	- 0·234
E	- 4,363·013	—	+ 0·084	+ 0·007
F	+ 2,272·694	—	+ 0·156	- 0·059
G	+ 9,453·586	—	- 0·644	+ 0·214

The significance of the additional variable is that the regression technique claims to have found some underlying cyclical pattern in the data which has a greater statistical significance than the named variables, so the model has failed to supply a satisfactory explanation of the data. The parameters arrived at in this calculation are not very sensible; however they are the linear functions which produce the 'best fit' between the inputs and the outputs in terms of the given maintained hypothesis, as a result of using this regression technique! In this case the econometrician would not be able to obtain reduced-form parameters directly, and would have to construct a set of more detailed structural equations, before attempting a further regression.

2.8 THE PHYSICAL MODEL

The third alternative method of quantification involved consideration of the chemical, biological or physical changes which the processes are bringing about in the material passing through them. The econometric model considers the inputs and the outputs and seeks to link them by statistical inference; one could call this modelling 'from the top downwards'. The technological model builds 'from the bottom upwards' with the detailed standard usages and costs of the various physically discernible operations. Both ignore the fact that in many manufacturing processes the input-output yield over very lengthy series of operations could be calculated in advance from a knowledge of the chemistry, etc. involved, without a need for observations of any kind. It should be remarked that this fact will be of assistance to the model builder only when it is physically or commercially impossible for the material to follow alternative paths through the system. If Product M can either be sold externally or processed further within the system to make Product N, one cannot look upon the manufacture of N as some chemical reaction on the material passing through M, since the latter may not all pass on to become N.

It is necessary to return to the definition given earlier of the term 'process', since many industries contain examples of series of distinct manufacturing operations, all of which have single homogeneous outputs (or parcels of outputs), but which cannot be linked to their neighbours in more than one way. These industries, or large sections of them, might be looked upon as *single* processes throwing off large numbers of joint products at various points within themselves. The various products may be thrown off at the end of different operations, but always in invariant proportion one to another, since the basic throughput of the series of operations is constant. This is the situation in which models of the third type can be built, and it can be seen that these models simply calculate the input coefficients for a single 'process', as we usually define them. A fairly well-known example of the technique is to be found in the 'Pichler model' (89 and 90), which also shows how the input-output coefficients of a process can vary without any technological change having occurred.[18] This can result from differences in the temperature and humidity under which the process operates, the time allowed for the reactions to occur, and the purity of the input material. Considerations of this type are especially important in joint production, since it makes it possible to vary the proportions of the outputs to some extent.

It is not feasible to construct a Pichler model for the system shown in Figure 2.1 and further described in the preceding paragraphs, because all seven products have been assumed to be externally saleable. However the following example uses the same flow chart but assumes that only A and E are saleable, and C and D are the two raw materials used to manufacture A, while D alone is used to make E. This means that two distinct processes are being modelled here; one for the manufacture of A (comprising sub-processes B and A) and the other for E (comprising sub-processes G, F and E). As has been pointed out, the technique can consider a process which manufactures several products in joint production but cannot construct a model to contain separately variable outputs. Assume that the chemical reactions occur in the course of the processes (using an invented 'Mickey Mouse' chemistry!):

(A) The A-process chain

(i) Sub-process B converts 700 units of a gas D (X_2O) plus 400 units of a gas C (pure X) into 800 units of C; 200 units of D remain unconverted in the mixture and there is also a loss of

100 units of O from D in the sub-process. This imaginary chemical process reflects the inputs shown on Figure 2.1:

Gas	Input (i) in thou. cu. ft.	Yield (y) in thou. cu. ft.
C	0·4 (0·4 X)	0·8 (0·8 X)
D	0·7 (0·56 X + 0·14 O)	0·2 (0·16 X + 0·04 O)
	1·1	1·0

The Pichler model expresses this operation basically as a set of simultaneous equations:

$$c = \quad c + \frac{4}{7}d$$

$$d = -c + \frac{6}{7}d$$

This is then restated as a 'transformation matrix' ($[U^b]$), which will convert the input vector into the yield vector:[19]

$$\begin{array}{c} \\ c \\ d \end{array} \begin{array}{cc} \multicolumn{2}{c}{[U^b]} \\ c & c \\ 1 & \frac{4}{7} \\ -1 & \frac{6}{7} \end{array} \times \begin{array}{c} [i^b] \\ 400 \\ 700 \end{array} = \begin{array}{c} [y^b] \\ 800 \\ 200 \end{array}$$

(ii) Sub-process A merely doubles the quantities of both C and D (say by decompressing it to the level of the atmosphere), and the transformation matrix $[U^a]$ is:

$$\begin{array}{c} \\ c \\ d \end{array} \begin{array}{cc} c & d \\ 2 & 0 \\ 0 & 2 \end{array} \times \begin{array}{c} [i^a]\ (\equiv[y^b]) \\ 800 \\ 200 \end{array} = \begin{array}{c} [y^a] \\ 1{,}600 \\ 400 \end{array}$$

It is now possible to arrive at an overall transformation matrix for the whole process, $[U^A]$, by multiplying together $[U^b]$ and $[U^a]$; this will then convert the original input into the final yield of the process:

$$
\begin{array}{c|cc}
 & \multicolumn{2}{c}{[U^A]} \\
 & c & d \\
c & 2 & \frac{8}{7} \\
d & -2 & \frac{12}{7}
\end{array}
\times
\begin{array}{c}
[i^b] \\ \\
200 \\
350
\end{array}
=
\begin{array}{c}
[y^a] \\ \\
800 \\
200
\end{array}
$$

The original input has been halved in this calculation, simply to show that the result coincides with that shown by the [*T*]-matrix of the technological model; namely that to make one unit of Product A requires inputs of 0·2 unit of C and 0·35 unit of D.

(B) The E-process chain

(i) The sub-process G simply (say) compresses chemical D by one-sixth; an input of 1440 cu. ft. produces a yield of 1200 cu. ft., and no matrices or vectors need be constructed.

(ii) Sub-process F takes the 1200 cu. ft. of compressed D from sub-process G and adds a further 450 cu. ft. of uncompressed D to make 1500 cu. ft. of some mixture of the two. A simple process might allow the uncompressed D to pass through unchanged while losing one-eighth of the compressed D, resulting in this transformation:

$$
\begin{array}{c|cc}
 & \multicolumn{2}{c}{[U^f]} \\
 & d^g & d^f \\
d^g & \frac{7}{8} & 0 \\
d^f & 0 & 1
\end{array}
\times
\begin{array}{c}
[i^f] \\ \\
1{,}200 \\
450
\end{array}
=
\begin{array}{c}
[y^f] \\ \\
1{,}050 \\
450
\end{array}
$$

(iii) Sub-process E compresses the mixture by one-third, as

$$
\begin{array}{c|cc}
 & \multicolumn{2}{c}{[U^e]} \\
 & d^g & d^f \\
d^g & \frac{2}{3} & 0 \\
d^f & 0 & \frac{2}{3}
\end{array}
\times
\begin{array}{c}
[y^f] \\ \\
1{,}050 \\
450
\end{array}
=
\begin{array}{c}
[y^e] \\ \\
700 \\
300
\end{array}
$$

If a notional transformation matrix was to be constructed for sub-process G, the final overall transformation matrix, $[U^E]$, could then be arrived at in the normal way.

2.9 SELECTION OF A METHOD

The technological model will usually need a much larger matrix than either the econometric model or the physical model; however it is very flexible in the face of technological change, since the coefficients of $[\mathcal{N}]$ can be amended to cope with changes in the yield, while a change in the structure itself can be effected by adding or deleting the relevant columns or rows. In contrast the possibility of making any changes at all in the econometric model would depend upon the detail and accuracy of the structural equations. The physical model would need to be reconstructed entirely to handle a change in structure, although changes in yield could be entered very easily in the appropriate transformation matrices. As pointed out at the end of the previous chapter, with respect to the accounting system itself, the choice of the underlying 'model' also depends upon one's expectation of change. If neither the yields nor the structure is expected to change, the econometric model will probably provide a highly economical method; if the yields can change, but no great alterations are expected in the structure itself, the physical method would prove useful. However where both yields and structure are liable to change, the greater flexibility of the technological model is needed.[20]

It might be helpful to explain exactly the connection between the accounting system and its 'model' of the underlying real system. Whichever method of quantification is used, the list of variables in the model, both exogenous and endogenous, will provide the chart of accounts. The manual of procedures which indicates the links within the structure will be the $[\mathcal{N}]$-matrix in the technological model; the econometric and physical models are concerned with mapping exogenous inputs directly into the net outputs of the system, so their manuals of procedures will contain material akin to $[T]$, in that it will reflect the full ancestry of the products rather than their next-assembly relationships.

2.10 NOTES

1. An even more comprehensive treatise on this topic is Rivett's *Principles of Model-Building* (94). This work is particularly to be commended for its continual emphasis that no degree of mathematical sophistication will overcome an essential naïvety in the basic structure of the model.

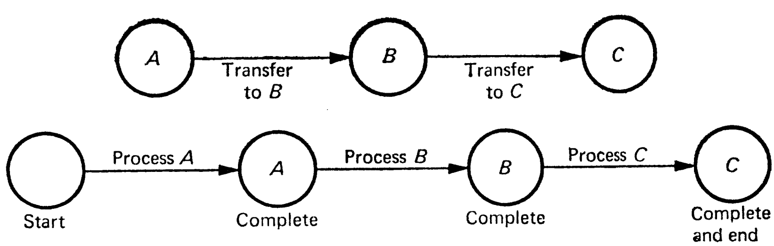
2. This chapter does not propose to enter into the rightly debatable problem of how 'standards' should be set; interested readers are referred to Stedry (104). However it will be necessary to return to the matter in the latter part of the book. Also we are discussing the 'technological model' as though it were just a method of quantifying an *a priori* structural equation, involving the use of standard costs as opposed to statistical inference. Obviously things could be done in this way. However it will be apparent from succeeding sections of this chapter that 'technological model building' as usually practised dispenses with any *a priori* equation and ferrets out its own structure from the basic standard costing data provided.

3. A paper by Paelinck and De Boer (88) has been drawn to my attention as a Dutch contribution to this field. There are at least two other papers on public record which are of a very different nature: Noble (85) and Tuckett (108) describe practical applications of this technique in the chemical industry.

4. Again, it is not proposed to explain the differences between the two methods in this book, since the general reader would find it confusing if another technique were to be described, only to be rejected for this purpose. Interested readers are referred to the paper by Gambling and Nour which sets out the arguments (44). This paper was the subject of a 'Comment' (101) and a 'Reply' (45) which may help to explain the issues quite fully.

5. Graph theory is well covered by Berge (10); the persevering reader will observe many parallels between what is done in that book and other branches of science and engineering. The extension of these ideas to economic material is to be found in Ford and Fulkerson's work (39), while Seppánen and Moore attempt to apply graph theory to more general problems in management science (100).

6. One might wonder how this 'network' relates (if at all) to the more familiar PERT/CPM network. The gozintograph designates the nodes as 'processes' and shows the input per unit of output as the arcs. By contrast the PERT diagram shows the processes are arcs and the nodes designate the completion of one process and transfer to the next; thus the PERT diagram is as it were a photographic negative of the gozintograph:

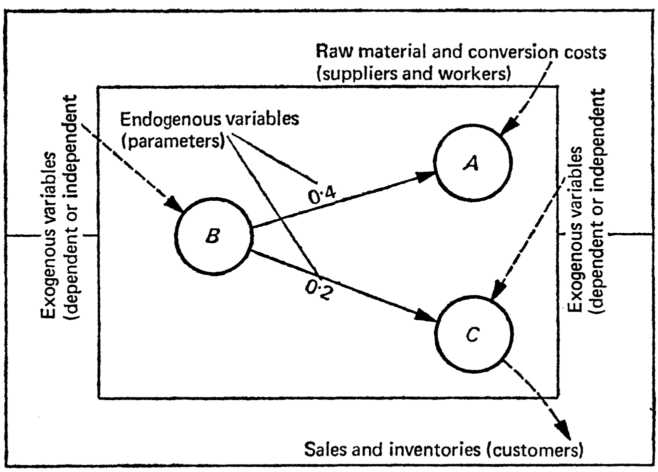


The extension of PERT to cover cost and similar matters are discussed by De Coster (26) and also by Charnes and Cooper (20).

7. Any textbook with a title like 'Mathematics for Managers' or 'Mathematics for Engineers' will contain a section on matrix algebra. They should be approached with caution, since the methods they describe are too cumbersome to deal with matrices of $10{,}000^2$ items; this book will suggest alternative, practical methods as the need arises!

8. It is important not to confuse 'endogenous' and 'exogenous' with concepts such as 'dependent and independent variables' or 'open and closed systems.' Since all processes must involve a conversion cost even if they do not require the introduction of additional raw material, micro-systems will always be open; what is dependent or independent will follow from what is considered to be known or unknown. Figure 2.13 may help.

FIGURE 2.13



The raw material suppliers, etc. are dependent on the activity of the system for the demand made on them but independent as to the prices charged by them: thus they are independent variables if one wants to know 'what are the variable unit costs of the products?', but dependent variables if the question is 'What will be the total activity in process X, given this required vector of net output?' Much the same in the reverse direction can be said of the customers. It can be seen that the endogenous variables provide the parameters of the model itself: these are essentially independent.

9. One should note that a mathematician would say that the matrix in question had a triangular form anyway! We are putting it into a *visibly* upper-triangular version for ease in illustrating the technique of sequential inversion only. It is also important to realise that one would never rearrange the matrix in the computer itself! All that we need is a sort of index to find our way around it. Readers who consult Vazsonyi's seminal paper

(op. cit.) may find it rather heavy going because, in effect, it uses such an index rather than a visually apparent order. 'Triangularisation' is a familiar term to macro-users of input-output models; it is used to try and establish an inter-industry hierarchy showing primary and subsequent producers, e.g. Korte and Oberhofer (69).

10. It should be observed that Wenke's paper and the algorithms it contains relate only to triangularisation. The subsequent inversion process is quite separate from it. Indeed it will be seen in Chapter 3 that inversion is not always needed to prepare the cost model, provided the triangular sequence is known.

11. Note that this is a 'reduced *relationship* matrix'; it is not the 'reduced *form*' of the technological model itself. The latter phrase is mostly used in connection with the econometric model, as we shall see in Section 2.7. Once again, the actual $[\mathcal{N}]$-matrix remains unaltered while the reduction algorithm proceeds, and the rows and columns are removed in this illustration only for the sake of clarity of exposition. Of course properly speaking, this (node to node) relationship matrix shows 1 where a direct relationship exists between two nodes and zero elsewhere (see Berge op. cit.). Thus the layout is identical with that of $[\mathcal{N}]$ where,

$$r_{ij} = \begin{cases} 1, & n_{ij} > 0 \\ 0, & n_{ij} = 0 \end{cases}$$

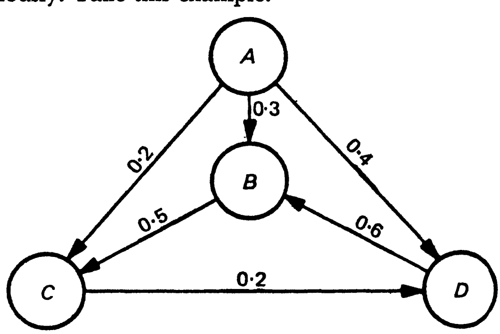
Obviously in practice one would simply use the $[\mathcal{N}]$-matrix itself, but it is better to use a separate matrix for illustration, to emphasise that $[R]$ is *not* $[\mathcal{N}]$.

12. The final section of Equation (2.10) is simply the formula for the multiplication of matrices; matrix $[T]$ is multiplied by the first $(j-1)$ items of the jth column in $\mathcal{N}$ to produce the first $j-1$ items of the jth column of $[T]$. 1 is then added as t_{jj} and the necessary zeros entered as $t_{j1} \ldots t_j\ (j-1)$ to complete the new (square matrix) $[T]$. The process is then repeated, and so on.

13. This involves the following rather fearsome notation

$$t_{ij} = \sum_{\iota=a}^{\iota=\nu} n'_{i\iota} \cdot t_{j\iota} \tag{2.12}$$

where a and ν are respectively the (arbitrarily selected) 'first' item in the circuit and the 'last' item; n' is used to show that the full ancestry has been entered previously. Take this example.



The approximate [*T*] of the circuit itself ('the lightly shaded area') taken to three places of decimals only, is:

	B	C	D
B	1·064	0·128	0·640
C	0·533	1·064	0·320
D	0·106	0·213	1·064

so 'the heavily shaded area' for the sole ancestor A becomes:

	B	C	D
A	1·064 × 0·3 = 0·319	0·533 × 0·3 = 0·160	0·106 × 0·3 = 0·031
	0·128 × 0·2 = 0·026	1·064 × 0·2 = 0·213	0·213 × 0·2 = 0·043
	0·640 × 0·4 = 0·254	0·320 × 0·4 = 0·128	1·064 × 0·4 = 0·426
	0·599	0·501	0·500

The resulting [*T*]-matrix shown on page 59 tells us that a throughput of 1064 items is needed in processes B, C or D to produce 1000 items for exogenous consumption.

14. A most interesting paper by Elliott and Uphoff (33) explores the possibility of using this technique for short-range profitability forecasting. Since their model considers the effect of marketing variables it obviously possesses the 'black box' features which make the method most suitable. They comment on the superiority of their results to those obtained by various extrapolation techniques: this point will be reviewed in Chapter 4.

15. The econometrician T.-C. Liu has pointed out (75) that the preliminary structural estimation is often very difficult in practice. He has claimed also that it is rather dangerous to place *a priori* restrictions upon the variables to be used in arriving at the reduced forms, since it is likely that a model builder will incorrectly specify the variables which are exogenous to the system.

16. These costs have been arrived at by use of the cost-modelling technique to be described in detail at the beginning of Chapter 3. The point to note is that the underlying model not only totally explains the movement of the figures, but it is strictly linear as well. The total costs for each process have been split between X, Y and labour in varying proportions.

17. This hypothesis is very nearly 'correct', except that C also uses a small quantity of Y. The significance of this exercise is that multiple regression does not seem to come very close to finding a sensible set of parameters for a very straightforward problem. It would be interesting to conduct a series of experiments to see how different misspecifications (or even the correct one!) or more 'sophisticated' regression techniques affected the validity of the results obtained! A paper by Benston (9) explores the use of the techniques of multiple regression in cost ascertainment.

18. Obviously engineering journals will abound with models of this general

	B		C		D	
	Activity in circuit	Use of A	Activity in circuit	Use of A	Activity in circuit	Use o A
Start	1,064		1,064		1,064	
Sell	1,000		1,000		1,000	
Surplus in production	64		64		64	
Which makes	(1/0·5) 128C	(0·2) 25	(1/0·2) 320D	(0·4)128	(1/0·6)106·6B	(0·3) 31
Which makes	(1/0·2) 640D	(0·4)256	(1/0·6) 533B	(0·3)160	(1/5) 213·3C	(0·2) 43
Which makes	(1/0·6)1,066B*	(0·3)319	(1/0·5)1,066C*	(0·2)213	(1/0·2)1,066D*	(0·4)426
		600*		501*		500*

* The errors follow from the rounding-off operation. However one must be wary of 'close approximations', where circuits are involved. A discussion paper by Gambling and Rickwood explores the problems of approximations of inverses in general (46).

type. However our engineering colleagues do not too often concern themselves with the costing implications in the way that Pichler does. An interesting example of another model with financial interfaces is in a paper by Tihanyi and Tihanyi (106); the journal *The Engineering Economist* is in general 'a good buy' for this sort of material.

19. The technique is (a) to complete the diagonals by considering whether any loss or gain is occurring in the sub-process and then (b) showing how the various elements are taken up or lost between the compounds (the proportions for any one compound are always calculated by reference to the bulk of the *other* compound involved in the transfer).

20. The choices between the models can be illustrated by the following schema:

Model	Basic technique	Assumption about the underlying reality	Reality best served
Technical model	Standard costing	A directly observable phenomenon which cannot be guaranteed not to change its method of operation	Micro-aspects of human activity (subject to negotiation)
Econometric model	Statistical inference	Not directly observable – a 'black box'	Macro-aspects of human activity
Physical model	Theory of physics or chemistry	A directly observable phenomenon which cannot function in any other way	Replicable, non-human activity

3 The Model in Action

3.1 THE NORMAL COST OF MULTI-PROCESS PRODUCTION

This chapter shows how the models which have been described are used in the management of the enterprise. Firstly, it is necessary to consider whether it is quite essential to know the total cost *in all processes* of any one product. The question is not usually asked, since traditional accounting methods require a value, basically the total manufacturing cost, to be placed upon the outstanding inventories at the end of the accounting period. In Chapter 1 it was explained that this method of arriving at the figure of profit is of questionable accuracy, and another method was outlined which was independent of historical cost. From this it would appear that it is not necessary to ascertain the total costs of any one product, provided that the future cash flows can be calculated solely by reference to the expected gross output of the various processes; this will usually be the case.

Apart from their use in final accounting procedures, the models also form the basis for any medium-term planning exercises which the enterprise may undertake. Something more will be said about this aspect of their use in the next section of this chapter, but in the meantime it is necessary to refer briefly to the most likely of the planning techniques, mathematical programming, since this also might seem to depend upon the ascertainment of total cost. It is not proposed to explain the conventional techniques of mathematical programming in this book; it is therefore presupposed that the reader does possess a basic understanding of the techniques of programming. The usual presentation of a linear programming problem as a solution to a budgeting problem, takes the net 'contribution'[1] of the various products as the 'objective function' associated with the columns representing the output of the various processes. Nevertheless this presentation is not essential, provided that additional columns are added to the matrix to represent the sales of the products; the net sales incomes can then be attached to the latter in the objective function, while the process output columns carry their unit variable conversion costs as negative contributions. These alternative treatments will be discussed later in the chapter.

Since the accounting system discussed in Chapter 1 is essentially a standard marginal costing system, these total unit costs can be described as the 'standard variable' or 'normal' cost of the product. Although both the profit and the optimal budget can be calculated without knowing what the normal costs are, these will almost always be calculated for other reasons. One is that the matrix is already inconveniently large and in general one would not want to add further columns or rows to the programming problem. Another is that some firms may be in a position to fix prices to some extent, when it will be important to know what costs are incurred in manufacturing the individual products.

Once the basic technological matrix [*N*] has been placed into triangular order, the normal costs can be calculated directly from this without ascertaining [*T*]. Returning to the example set out in Figure 2.1, the triangularised form of [*N*] (which has not hitherto been illustrated directly) is illustrated by Figure 3.1.

FIGURE 3.1

		Outputs						
		D	G	C	B	F	A	E
	D		1·2		0·7	0·3		
	G					0·8		
	C				0·4			
Inputs	B						0·5	
	F							1·5
	A							
	E							

The standard unit conversion costs (exogenous costs) in each process are given as:

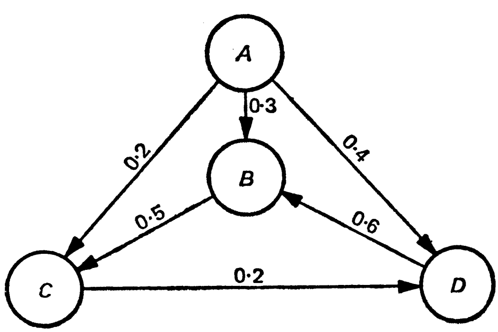
Process	A	B	C	D	E	F	G
Cost	£3·00	£5·00	£7·00	£6·00	£1·00	£2·00	£4·00

Since the triangular form ensures that any endogenous inputs will have been fully evaluated already, the calculation of the normal cost of the product is:

Product	Exogenous costs £	Endogenous costs	£	Normal cost £
D	6·00		—	6·00
G	4·00	(D) 1·2 × 6 =	7·20	11·20
C	7·00		—	7·00
B	5·00	(D) 0·7 × 6 = (C) 0·4 × 7 =	4·20 2·80	12·00
F	2·00	(D) 0·3 × 6 = (G) 0·8 × 11·20 =	1·80 8·96	12·76
A	3·00	(B) 0·5 × 12 =	6·00	9·00
E	1·00	(F) 1·5 × 12·76 =	19·14	20·14

This technique will also work for a circuit, although this cannot be placed into triangular form. Consider the gozinto-graph which has already appeared in Note 13 of Chapter 1 (see Figure 3.2).

FIGURE 3.2



where $[N]$ is therefore:

	Outputs			
	A	B	C	D
A		0·3	0·2	0·4
B			0·5	
C				0·2
D		0·6		

and these are the standard unit conversion costs

A	B	C	D
£100	£10	£1	£5

It is apparent that the normal cost of Product B cannot be calculated before that of Product D, which is one of its inputs; similarly Product D requires the previous calculation of a cost for Product C, which in turn depends on Product B. This can be resolved by considering the results of acting upon the following assumptions:

(a) – that the normal cost of Product D is nil, and
(b) – that the normal cost of Product D is £100.

Now if (a) were true the calculation would run:

Product	Exogenous costs	Endogenous costs	Normal cost
	£	£	£
A	100	—	100·00
B	10	(A) 0·3 × 100 = 30 (D) 0·6 × 0 = 0	40·00
C	1	(A) 0·2 × 100 = 20 (B) 0·5 × 40 = 20	41·00
D	5	(A) 0·4 × 100 = 40 (C) 0·2 × 41 = 8·20	53·20

While if (b) is true the normal costs become:

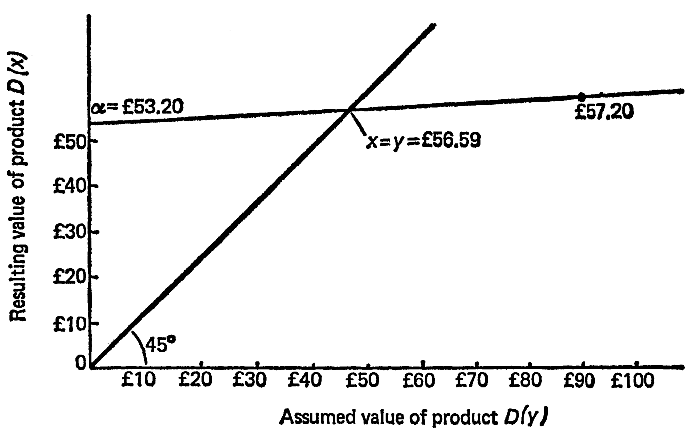
Product	Exogenous costs	Endogenous costs		Normal cost
	£		£	£
A	100		—	100·00
B	10	(A) 0·3 × 100 = 30 (D) 0·6 × 100 = 60		100·00
C	1	(A) 0·2 × 100 = 20 (B) 0·5 × 100 = 50		71·00
D	5	(A) 0·4 × 100 = 40 (C) 0·2 × 71 = 14·20		59·20

It is apparent that neither assumption is true! However it is possible to use these two calculated values of Product D to arrive at the true value; this is because the basic assumption that there are constant variable conversion costs per unit of output is an assumption that the cost function is linear. Now the formula for a linear function (or straight line) is:

$$x = \alpha + \beta y$$

where α is its starting point on the upright axis of the graph, and β is the gradient at which it climbs. If y is taken to be the assumed value of Product D, the calculations have established two points on Figure 3.3.

FIGURE 3.3



If a line is extended at 45 degrees from the origin of the axes, it will cut the line $x = \alpha + \beta y$ at the point at which $x = y$; this is the true cost of Product D, since it is the only value which will reproduce itself as the result when used as an input into the calculation. It is not necessary to draw the graph itself, since α and β can be calculated from the 'sighting shots' themselves.

α is clearly the value of Product D arrived at when $y = 0$; that is to say £53·20.

β, the gradient, is the difference between the two results, divided by the difference between the sighting shots; here (59·20 − 53·20)/100 or 0·06.

The true value is arrived at as:

$$\frac{\alpha}{1-\beta} = \frac{53{\cdot}20}{0{\cdot}94} = £56{\cdot}59$$

The calculation can then proceed as:

Product	Exogenous costs	Endogenous costs		Normal cost
	£		£	£
A	100		—	100·00
B	10	(A) 0·3 × 100 = 30 (D) 0·6 × 56.59 = 33·93		73·93
C	1	(A) 0·2 × 100 = 20 (B) 0·5 × 73.93 = 36·96		57·96
D	5	(A) 0·4 × 100 = 40 (C) 0·2 × 57.96 = 11·59		56·59

It should be observed that the more complex circuits shown in Figure 2.7 might be rather more difficult to resolve in this way. In such cases it would be better to calculate the normal costs from [*T*] instead of [*N*]. To do this each row of [*T*] needs to be multiplied by the appropriate standard unit conversion cost of the process; the columns can then be summed to produce the normal costs. This technique would arrive at the same results for the examples calculated above from [*N*].

3.2 ACCOUNTING WITHIN THE MODEL

Once [$\mathcal{N}$] has been constructed and the standard unit conversion costs ascertained, the accounting procedures themselves are entirely straightforward; the actual conversion costs and endogenous usages of the various cost centres are compared with the standard costs and usages of their actual outputs.[2]

Returning to the example from Figure 2.1 and assuming that each process was carried out exclusively in a single plant, each consisting of one cost centre, these actual results have occurred in some accounting period (of 30 days):

Product/ Process	Actual gross output (units) (a)	Actual endogenous transfers (units) (b)	Actual net outputs (units (c)	Actual conversion costs[3] £ (d)
A	3,700	—	3,700	11,000
B	2,900	1,900 to A	1,000	15,000
C	6,450	1,450 to B	5,000	43,500
D	14,450	1,750 to B; 800 to F; 5,900 to G	6,000	84,000
E	1,500	—	1,500	1,550
F	4,300	2,100 to E	2,200	8,750
G	6,000	3,500 to F	2,500	23,800
				£187,600

'Standard' data can be worked out for the period in the usual fashion.

Product/ Process	Standard conversion cost of actual gross output £ (e)	Standard endogenous inputs for actual gross output (units) (f)	Total normal cost of actual net output £ (g)
A	11,100	1,850 (B)	33,300
B	14,500	1,160 (C); 2,030 (D)	12,000
C	45,150	—	35,000
D	86,700	—	36,000
E	1,500	2,250 (F)	30,210
F	8,600	1,290 (D); 3,440 (G)	28,072
G	24,000	7,200 (D)	28,000
	£191,550		£202,582

There is a simple expenditure variance between the actual and standard conversion costs ($d - e$):

Process	Variance
A	100 favourable
B	500 adverse
C	1,650 favourable
D	2,700 favourable
E	50 adverse
F	150 adverse
G	200 favourable
	£3,950 favourable

Although it adds little to the control of the operation, it would be possible to evaluate the variances in endogenous inputs also, at the normal cost of the input.

Process	Endogenous input variance in units (b – f) £	Endogenous input variance at normal cost £
A	+ 50 (B)	600 adverse
B	+ 290 (C)	2,030 adverse
	– 280 (D)	1,680 favourable
E	– 150 (F)	1,914 favourable
F	– 490 (D)	2,940 favourable
	+ 60 (G)	672 adverse
G	– 1,300 (D)	7,800 favourable
		£11,032 favourable

Presumably it would be possible to analyse all these variances further in the traditional fashion, but these analyses will not contribute much to the control of the system. The figures calculated here will explain the difference between the actual conversion cost (a) and the normal cost of the net output (g):

	£
Actual conversion cost	187,600
add favourable net cost variance	
on exogenous items	3,950
on endogenous items	11,032
Normal cost of net output	£202,582

The fact that economic accounting is based upon cash flows means that it will be possible to calculate a further variance which is not available to traditional standard costing procedure. Cash budgeting requires the establishment of budgeted collection periods (and other periods for credit) and inventory levels; this means it would be possible to arrive at a 'cash collection variance', which would be the difference between the 'standard' working capital items at the end of the period (calculated from the accrual-based conversion costs) and their actual amounts. This variance will explain the difference between the actual cash flow and the standard flow for the accrual-based conversion costs for the period. If some standard selling prices are assumed for the products and it is assumed that no sales price variance or change in the finished goods inventory has occurred, the actual (and normal) sales become:

Product	Price	Normal sales
	£	£
A	12·00	44,400
B	16·00	16,000
C	9·00	45,000
D	9·50	57,000
E	26·00	39,000
F	17·00	37,400
G	14·00	35,000
		£273,800

If the actual opening debtors were £180,000 and the closing debtors were £113,800, the actual positive cash flow would have been £340,000. Supposing the standard collection period was twelve days (and the sales are assumed to occur regularly throughout the period), the standard debtors would be only (12/30 × 273,800) £109,520, so that the standard positive flow would be £344,280.

Similarly for the negative cash flow. Starting with the actual conversion costs the calculation of the actual flow would be as shown on page 70.

If the standard closing creditors and inventories were respectively 25 per cent and 50 per cent of the actual conversion costs (£46,900 and £93,800), the standard negative cash flow

	£
Conversion costs	187,600
add commencing creditors	50,000
	237,600
less commencing raw material inventories	120,000
	117,600
less closing creditors	40,000
	77,600
add closing inventories	100,000
	£177,600

would become £164,500. The analyses could be summarised as:

	£	£
Actual net cash flows (340,000–177,600)		162,400
add cash collection variances		
Debtors	4,280	
Creditors	6,900	
Inventories	6,200	17,380
Standard net cash flow (344,280–164,500)		£179,780

3.3 THE [T]-MATRIX AS AN L.P. PROBLEM

The contributions for the products shown in Figure 2.1 are:

	A	B	C	D	E	F	G
	£	£	£	£	£	£	£
Net sales income	12·00	16·00	9·00	9·50	26·00	17·00	14·00
Less: Normal cost contributions	9·00	12·00	7·00	6·00	20·14	12·76	11·20
	3·00	4·00	2·00	3·50	5·86	4·24	2·80

If some capacities are also assumed for each of the processes[4] in the system, these contributions can now be combined with them and the [T]-matrix calculated in Chapter 2, to form a normal linear programming problem:

Maximise $3{\cdot}00a + 4{\cdot}00b + 2{\cdot}00c + 3{\cdot}50d + 5{\cdot}86e + 4{\cdot}24f + 2{\cdot}80g$

Subject to:

$$
\begin{array}{lllllllll}
\text{(Process A)} & a & & & & & & \leq & 5{,}000 \\
\text{(Process B)} & 0{\cdot}50a + & b & & & & & \leq & 5{,}000 \\
\text{(Process C)} & 0{\cdot}20a + & 0{\cdot}40b + & c & & & & \leq & 8{,}000 \\
\text{(Process D)} & 0{\cdot}35a + & 0{\cdot}70b & & + d + & 1{\cdot}89e + & 1.26f + & 1{\cdot}20g \leq & 24{,}000 \\
\text{(Process E)} & & & & & e & & \leq & 2{,}000 \\
\text{(Process F)} & & & & & 1{\cdot}50e + & f & \leq & 5{,}000 \\
\text{(Process G)} & & & & & 1{\cdot}20e + & 0{\cdot}80f + & g \leq & 7{,}000
\end{array}
$$

In most practical instances further constraints will also be needed; for example the quantities which can be sold at the given prices are likely to be limited. Since each row in the tableau set out above (except the first, which is the 'objective function') is an inequality which constrains outputs within the capacity of one of the various physical processes only, it is necessary to add further rows to take account of these. Thus the market constraints take the form of 'upper bounds' on the variables, and might be:

$$
\begin{array}{llllllll}
a & & & & & & & \leq 4{,}500 \\
 & b & & & & & & \leq 3{,}000 \\
 & & c & & & & & \leq 4{,}000 \\
 & & & d & & & & \leq 7{,}500 \\
 & & & & e & & & \leq 2{,}000 \\
 & & & & & f & & \leq 1{,}800 \\
 & & & & & & g & \leq 2{,}500
\end{array}
$$

Further rows might be added to deal with other limits which have to be placed on the operation of the system, such as finance. However it is not proposed to consider the normal simplex solution to this problem, since the real life problem is likely to be very much larger than this example. The largest linear programming problems directly solvable even by a computer seem to have involved about 4000 rows and half a million columns; it is the number of constraints which is usually critical rather than the number of variables. *Prima facie* the matrices being considered here have at least 10,000 rows, and so are infeasible for solution.

From what has been said here, it is clear that although it was suggested earlier that it might be acceptable to add additional *columns* to carry the selling prices of the products, while attaching negative objectives to the processes themselves, this alternative may not be advisable in a very large example. The additional columns will not matter in themselves and additional rows for the marketing constraints will be needed in any case; however it may be necessary to add further rows to constrain the sales within the net output of the processes.[5] Moreover, returning to Chapter 1, it may be easier to see the additional columns providing the chart of accounts for the analysis of sales, while the additional rows will reflect the manual of procedures for relating the cost-of-goods-sold to the relevant sales. Finally, under conditions of joint production, it is essential to proceed in this way, but care is necessary in using the variable standard costs per unit of the products which result from it, since their real marginal costs will depend upon the state of the demand for the other jointly demanded products.[6]

However for the purposes of this book it can be assumed that whatever steps are taken to avoid unnecessary increases in the size of the matrix, this will always be too large for direct solution by the normal simplex algorithm or any variant thereof. A number of techniques have been devised which can overcome this difficulty to some extent. One could construct some procedure which would approximate the linear programming solution, while being less prodigal in computational capacity – an 'heuristic' method. One could consolidate, or 'aggregate' the matrix, by locating and removing constraints which cannot for one reason or another become operative, or it may be possible to 'decompose' the problem into a master problem and a number of smaller sub-problems; these last devices would enable the normal programming algorithms to be used on the smaller problems, or sub-problems.

3.4 AN HEURISTIC SOLUTION

The simplex method of solving linear programming problems is 'algorithmic', which is to say that it will work away at the problems until it arrives at the absolutely correct, optimal answer. An alternative approach would be to take some feasible solution as a starting point and then 'search' to see if some slight adjustment to the solution would effect any improvement in the overall result; when no further improvements can be achieved, the process is concluded. This final solution may not be the optimal solution; the technique is sometimes called 'hill climbing' and the apparent final solution may just be a sort of shoulder on a much larger mountain of profit.[7]

The rules for an heuristic solution might be:

(a) Calculate the contribution to profit per unit sold of each product.
(b) Range the products in descending order of contributions per unit.
(c) Taking the product with the highest contribution, divide each input-output coefficient in its column in the [*T*]-matrix, into the appropriate constraints; this will show the largest quantity of the product which could be made with the amount of that resource which is available, and hence the resource which will prove to be its 'limiting factor'.
(d) If the limiting factor inhibits the production below the quantities demanded by the market constraints, the quantity to be produced will be the largest feasible amount.
(e) Calculate the requirements of the various resources needed to produce the quantity demanded (or calculated in (d)), and deduct these from the constraints, to leave the capacities available for further production of the remaining products.
(f) Repeat steps (c), (d) and (e) of this procedure with the next best products until they, or the constrained resources, are exhausted.

It can be seen that this procedure is identical to the initial steps in the simplex method of linear programming; the 'key column' and the 'key number' are ascertained and trial strategies are suggested. The difference is that no pivoting occurs about the key number, and once a product has been included in the trial strategy, it will never be displaced later by another product with a lower contribution, which nevertheless is less prodigal with resources required by other products. In fact, of course there is no facility here to calculate the 'dual' of the problem at all; no 'shadow prices' will be produced.

The process can be illustrated for the problem set out at the beginning of this section of the chapter.

The products are ranged as follows:

Order	Product	Contribution	Quantity demanded
		£	
1	E	5·86	2,000
2	F	4·24	1,800
3	B	4·00	3,000
4	D	3·50	7,500
5	A	3·00	4,500
6	G	2·80	2,500
7	C	2·00	4,000

(c) Taking product E the required inputs are:

(1) Resource	(2) Coefficient	(3) Constraint	(4) 3/2
D	1·89	24,000	12,698
E	1·00	2,000	2,000*
F	1·50	5,000	3,333
G	1·20	7,000	5,833

(d) Resource E is the limiting factor and is marked with an asterisk, but since only 2000 units are demanded, the sales budget remains feasible.

(e) The budgeting process can commence as:

(5) Make and sell	(6) Required resources (5 × 2)		(7) Remaining resources (3–6)
2,000 × E	D	3,780	20,220
	E	2,000	—
	F	3,000	2,000
	G	2,400	4,600

Product F must now be considered:

(c)

(1)	(2)	(3)	(4)
D	1·26	20,220	16,048
F	1·00	2,000	2,000*
G	0·80	4,600	5,750

(d) Resource F is the limiting factor, but again the sales budget of 1800 units remains feasible.

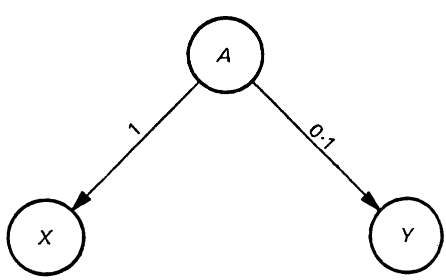
(e) The budget continues as:

(5)	(6)		(7)
1,800 × F	D	2,268	17,952
	F	1,800	200
	G	1,440	3,160

And so on; it is not proposed to reproduce the whole of the calculations necessary to arrive at the heuristic solution to this problem.[8]

It may be supposed that under appropriate circumstances a procedure of this sort will produce results very close to the true optimal contribution given by standard linear programming algorithms. The strategies suggested by the two methods may differ widely, but it seems that very little may be added to the quantity being maximised (or minimised) by the later reiterations of the simplex in these cases. Moreover the heuristic methods are comparatively simple to operate and can produce answers, even manually, to problems which would strain the resources of many computers to solve algorithmically. Nevertheless it is possible to produce other examples where this heuristic method will provide totally wrong results, so it is necessary to consider under which circumstances a product which is successful at an earlier trial should be displaced at a later stage. Clearly this may happen where a comparatively small quantity of a product with a larger contribution per unit, makes use of constrained resources so as to prevent the manufacture of much larger quantities of other products whose contribution per unit is smaller; in sufficiently extreme cases the *total* contribution from the latter may be greater than that from the former. In short outwardly less valuable products may show better returns on the scarce resources consumed.

Consider this simple example:



Product	Contribution	Constraint on Output
	£	Units
A	—	2,000
X	5·00	2,000
Y	2·00	20,000

Assume that any quantities of X and Y can be sold at prices which will produce these contributions.

The heuristic method just described would provide a solution of producing 2000 units of X; this would absorb the available capacity of A. As a result no Y could be made, and the consequent total contribution would be £10,000. The true simplex solution is quite to the contrary:

Budget X 0 units; Y 20,000 units
Total contribution £40,000
Opportunity cost of X £15 (20 − 5)
Shadow price of A £20

The maximum contribution available in this system is in fact £40,000; Y could afford to pay up to £20 for each unit of A so the effect of making a single unit of X is to lose a possible profit of £15. The reason for this reversal of the heuristic solution is the absence of constraint on the sales, which results in each 'trial' budget filling the network to the limit set by the limiting factor. This is less likely to occur: (a) where the demand is itself constrained by a sales forecast, and especially where the forecast demand for an otherwise homogeneous product is subdivided, with different prices and hence different contributions in a number of markets (this will tend to prevent any one product from flooding out the whole network when it is selected, and give some hope that a fair proportion of the optimal mixture will get through on the first iteration); and (b) where the capacity of the plant has been planned with a view to meeting the demands likely to be made upon it. Dramatic reversals in strategy can only occur where some resource is grossly overdemanded.

It might seem that both these conditions will almost always apply in a practical example, so that the heuristic method may have more to commend it than might appear from a more theoretical point of view.

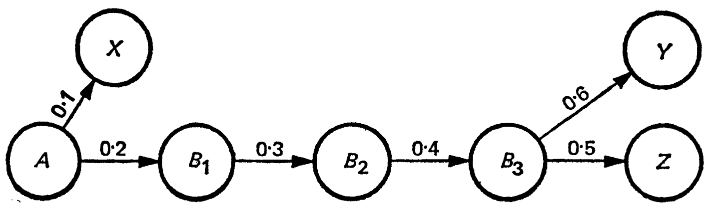
3.5 AGGREGATION OF THE TECHNOLOGICAL MATRIX

The second possible method for solving an L.P. problem whose basic formulation is too large for normal methods of solution, is simply to aggregate or consolidate the matrix, so that it becomes small enough for the purpose![9] A good deal has been written about aggregation and disaggregation in respect of macro-economic input-output tables, but the techniques to be described can usually be distinguished from this. In the previous chapter it was observed that discrepancies between the model and the underlying system were not of great moment provided that no technological change was occurring. Macro-economic input-output data are always aggregated to some extent, so those who use them must always ask what is the degree of the distortion introduced into the models by this fact. When items are aggregated together they are assumed to be entirely homogeneous; if A and B are aggregated into A/B, a demand for an A or a B are taken to be equally satisfied by an A/B. Similarly, the process which makes A and the process which makes B are supposed both to be able to make A/B's. Of course this is really a form of averaging, and the propositions will be strictly correct only to the extent that the individual products continue to be demanded or made equally. An alternative might be to make use of a weighted average, where the relative weightings are determined by the previous output or demand. However this device will continue to introduce distortion, since it assumes that the product will be made or sold in the same proportions in the future as in the past.

This distortion can only occur where both or all the processes to be consolidated in this way are 'economically significant'. This means that they must be either saleable independently, or usable by other products which are sold independently. When talking of aggregation of the technological model, only non-significant processes and products are considered to be capable of consolidation in this way, so this aspect of the problem does not arise. Nevertheless substantial savings can be effected by non-significant aggregation of this type, since the basic technological data available are rarely presented to the model builder in a consolidated form; instead the data will be ascribed to *visibly* independent processes of manufacture, whether or not they are capable of independent operation, let alone whether they will ever be required to operate other than at a single rate of throughput.[10]

Consider the example of Figure 3.4A.

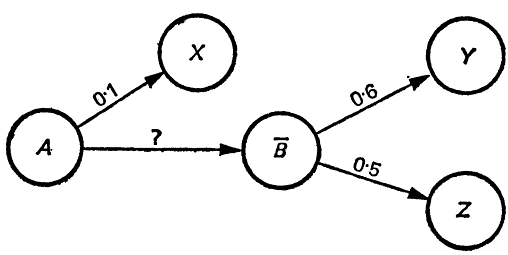
FIGURE 3.4A



(i) B_1, and B_2 and B_3 are not directly saleable; X, Y and Z are independently saleable. (ii) The capacities of the B processes are respectively 1000, 3000 and 8000 units. (iii) The unit variable conversion costs are A £100, B_1 £10, B_2 £30 and B_3 £40.

Clearly the real effect of this system is simply as pictured in Figure 3.4B

FIGURE 3.4B

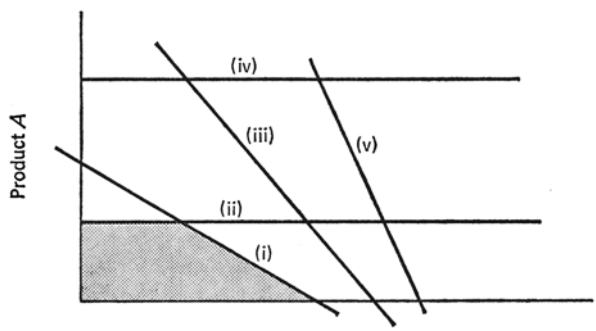


with the complication that the total unit conversion cost and capacity of $\bar{B}$ is not directly observable, and the input from A necessary to produce one unit of the 'product' $\bar{B}$ is also unknown. In fact the total conversion cost of $\bar{B}$ is £53·20, the capacity is 7500 units and the input required from A is 0·024 per unit. These figures have been calculated from the data of the individual processes using the methods which are described at some length in the author's thesis.[11]

However it is not proposed to reproduce the methodology here, because the [*T*]-matrix, and the normal costs, can all be readily calculated from the unconsolidated data; it is only to reduce the matrix for the purpose of optimisation that it becomes necessary to omit those *rows* which are held to be redundant. This redundancy may be inevitable, as in the example in Figure 3.4, where the constraints on B_1 and B_3 *never could* come into operation, but in a practical case it is usually possible to omit many other constraints even where they relate to processes whose products are economically significant. The author is indebted to Academician L. V. Kantorovich for the following example (in a conversation):

Assume a linear programming problem involving only two products, so that the solution can be shown in the form of a simple graph (Figure 3.5).

FIGURE 3.5



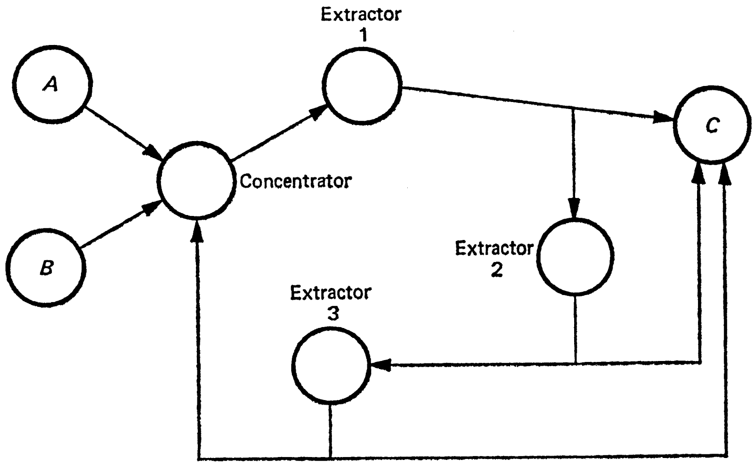
In a system of say 10,000 processes it is conceivable that 10,000 constraints (or more) *might* exist, but it is highly unlikely that more than 2000–3000 of them would be effective at any one time. In the figure there are five constraints shown, but only (i) and (ii) are effective in defining the area within which the feasible decisions lie. Moreover if the iso-profit line leaves that area at the point where (i) touches the B axis, it would not in the event be necessary to take account of the existence of (ii) either. The only problem is to locate these ineffective constraints in a massive system which is subject to continual change.

In the previous section of this chapter it was observed that while the academic examples often considered systems which were subjected to infinite demands, or demands which were widely at variance with their capacities, the contrary was likely to be true in practice. Clearly the plant has been constructed with a view to meeting the general pattern of demand, without undue waste. Thus it might be reasonable to take the previous period's sales and inflate them by, say, 20 per cent; these could then be used to multiply through the current [T]-matrix, to arrive at the gross outputs needed to achieve this large, across-the-board expansion. Those processes whose capacities were not infringed by so large an increase could then be assumed to be unlikely to affect the current solution, and be omitted accordingly. The solution resulting from the reduced problem would then be examined to see if in fact it infringes any of the constraints which were omitted; if so the problem would be reworked with those constraints reimposed.

It is possible to be beset by difficulties from a quite contrary cause. The technological data follow the visibly distinct operations; usually this will result in the provision of distinct data for a process which is not capable of independent operation, as in the previous examples. However it is possible that the data are themselves some sort of aggregate, which is not a matter of great importance so long as the sub-operations cannot vary their throughputs independently of each other. One might call this type of consolidation 'pre-aggregation': in practice it may cause considerable difficulties. Detailed examination of the sub-operations will often reveal that their throughputs are not strictly invariant, but rather that they have been assumed to be so, because a certain rate of throughput has been assumed to be optimal for the process as a whole. Consider the example of Figure 3.6 on page 81.

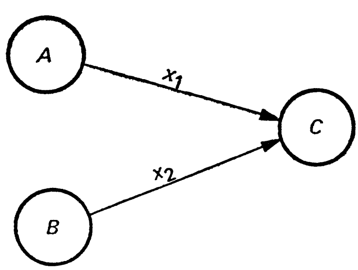
A chemical, A, and a solvent, B, are mixed together in a concentrator, and passed through a series of three extractor tanks; in each tank the chemical C is formed as a sediment. The reason for using more than one extractor tank is that the rate of sedimentation is not constant, but at first it occurs at an increasing rate and then at a declining rate. Finally the mixture is returned to the concentrator where it is restored to its original concentration and then recycled.

FIGURE 3.6



This example is adapted from one solved by Brosilow, Lasdon and Pearson (12). It is necessary to arrive at the 'optimal' periods for which the mixture should remain in each tank; if one believed that there was such a series of rates which would be optimal under all circumstances, this would dictate the speed at which the mixture flowed through the process, and hence the input-output coefficients, the variable processing costs per unit and the process's capacity in any period. That is to say, Figure 3.6 would become Figure 3.7.

FIGURE 3.7

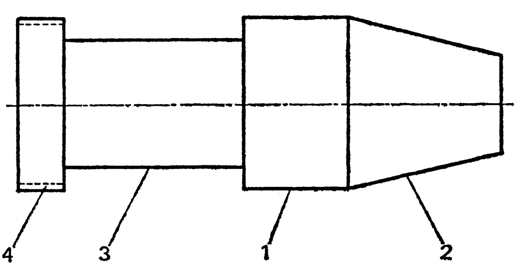


Presumably the optimal solution which would be used would be that which produced the chemical C at least cost. However if this process is just a sub-system which is part of a larger system, it is likely that any attempt to optimise the operation of the sub-system will sub-optimise the operation of the main system. Other parts of the system might be able to meet shadow prices in excess of the minimum cost of product C; unless the chemical could be obtained from outside the system for less than the shadow price offered, it would pay to expand the output of C at increased cost so long as it remained within the shadow price.[12]

The problem described above is of course a fairly difficult problem in non-linear programming, so one might feel that it was worthwhile to pre-aggregate the data in this way simply to be able to present it in the linear form of Figure 3.7. In any event it would be possible to solve the problem in this form, and make an *ad hoc* relaxation of the capacity if process C becomes a bottleneck. In general this sort of 'pre-optimisation' will often be employed, even without specific pre-aggregation, to avoid non-linearities; for example the speed of a machining operation is related to the angle of the cut of the tool, and so the standard time of an operation presupposes some 'optimal' combination has been selected in advance of the solution of the main problem.

A more subtle type of aggregation can sometimes be observed. Consider the component pictured in Figure 3.8, which might result from a series of operations carried out consecutively on a capstan lathe.

FIGURE 3.8



This illustration (adapted from one of Mitrofanov's illustrations in (78)) shows (1) an external cylinder, (2) an external cone, (3) an external undercut, and (4) an external thread. Presumably one could say that in particular circumstances it would take x minutes to machine this component. The capstan lathe was designed to obviate the need to change the cutting tool or to reload and unload the piece between operations, so there is little apparent need to consider the individual sub-operations. And yet a number of engineers, mainly in Germany and the Soviet Union, have pointed out that most of the components produced in any machine shop are very similar in design and manufacture, so that it is possible to assign a very large variety of products to a comparatively limited number of families or groups. This approach is known as 'group technology'.

Group technology makes possible certain economies in designing and manufacture, as well as uncovering possibilities for standardisation of the products. It may be possible, and preferable, to treat what is at present a series of apparently unconnected single jobs or small batches as flow-line or large-batch production of a single 'complex part', which incorporates all the internal and external shape elements which exist in one group of products. Further details of group technology and its applications would be out of place in a book concerned with accounting, model building and information systems; however it has been seen that technological data are assembled in the information system for general use within the organism, rather than for particular purposes. Returning to the discussion in Section 1.1 of this book, it may be that congruities exist between the sub-operations needed to produce the component in Figure 3.8 and those which manufacture other components. To present the data for the whole operation as a single item is to introduce an element of rigidity into the system which may not be justified.

3.6 DECOMPOSITION OF THE TECHNOLOGICAL MATRIX

The solution of very large systems of equations can also sometimes be effected by 'decomposing' the problem into a number of sub-problems, linked by a matter problem. The method is well known, and a number of techniques are adequately described in the literature (e.g. 24). It is not necessary to have any details of the constraints within the sub-problems in order to solve the master problem, so in most cases one might hope to reduce the large, protean problems under discussion to a number of separate problems, each of proportions within the capacity of the computer.

The protean, or rapidly changing nature of the technological model makes it as difficult to partition in this way, as it is to aggregate it formally. The reason is the same in each case; it is necessary to be able to detect the 'splits' or the 'chains' in the system without a physical, visible display. Algorithms have to be devised which would enable a computer to consider the available data sequentially, and decide whether or not these configurations exist in the system at the present time. Physical display would disclose the existence of sub-problems, if it were possible, since they will take the form of separate 'branches' in the tree-like graph formation referred to in the previous chapter. The basic example used in this book (Figure 2.1) exhibits this form; there are two distinct branches, C, B and A, and G, F and E, which are linked at D.

Unfortunately there is nothing in a random-ordered matrix for this graph which immediately indicates the possibility of partitioning the problem in this way. In particular the triangular form of the matrix which is arrived at by using the Wenke algorithms described earlier will not segregate the sub-problems (the products in Figure 2.1 are in fact ordered as D, G, C, B, F, A, E by this method). The reason is that the algorithm allocates the same γ-ratings to C and G, to B and F and to A and E. As was observed in the previous chapter, this order is not unique, since it is only the relative order of the directly related products which is significant for sequential inversion.

This difficulty could be overcome if the members of the various γ-groups are discriminated into their respective families, by requiring any two members of the same generation in the same family branch to have at least one common descendant. This can be ascertained from a search of the [T]-matrix; if two members of a generation are related in this fashion, at least one row of [T] will contain coefficients greater than zero in respect of both of them. It is clear that in this instance G and C have no descendants in common, so that their next common *ancestor* must be an articulation point in the graph. Examination of the rows of [T] then reveals that the 'families' of G and C are F and E, and B and A, respectively.

It is now possible to restate [T] for Figure 2.1 in a discriminative form:

	D	G	F	E	C	B	A	
D	1	1·2	1·26	1·89	0	0·7	0·35	Master problem
G		1	0·8	1·2				Sub-problem (a)
F			1	1·5				
E				1				
C					1	0·4	0·2	Sub-problem (b)
B						1	0·5	
A							1	

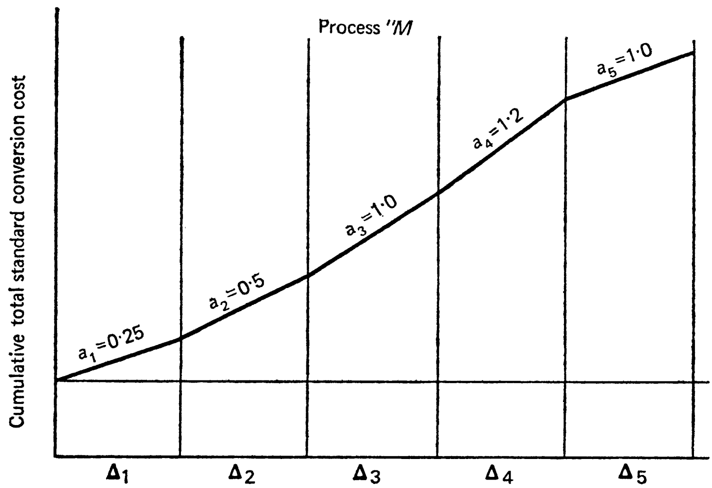
The partition is now apparent. It would be possible in a larger example to find more than one articulation point and so produce a series of sub-sub-problems nesting one within the other. Obviously the procedure would be to go to the earliest articulation point in the system and see whether it produced a master problem and sub-problem of convenient sizes, before considering further subdivisions.

3·7 SOME PROBLEMS OF NON-LINEARITY

Non-linearities in the input-output coefficients themselves are dealt with either by pre-aggregation (as discussed in Section 3.5) or by the familiar device of 'linear approximation', whereby the curved function is fitted to a series of straight lines or linear functions (see Figure 3.9).

The effect of this device is to divide the single 'real' process M into five (or more) sub-processes whose 'variable cost' is the appropriate a_i, but whose 'fixed costs' are the sum of $\Delta_1 a_1 + \Delta_2 a_2 + \ldots + \Delta_{(i-1)} a_{(i-1)} + b$. The outputs from all these 'sub-processes' need to be constrained within the capacity of the real process M. Obviously the sub-processes each produce a single homogeneous product variant, Product M_1, $M_2 \ldots$; the sales of these variants also require to be constrained within the demand for the single real product M. This example shows both decreasing returns to scale (for $\Delta_1 \ldots \Delta_4$) where no adjustment would be needed before using the resulting coefficients in a programming problem, since the least expensive variant will automatically be used first. For Δ_5 however the rate of growth of the costs is decreasing, so some special provision would be needed in the algorithm to prevent M_5 being used in the solution before M_4.

FIGURE 3.9



A rather more difficult problem is posed by the appearance of non-linearities in the objective function; for example:[13]

Maximise: $v_1 x_1 + v_2 x_2$

Subject to:

$$\begin{aligned} 0{\cdot}00400x_1 &+ 0{\cdot}00286x_2 \leq 100 \\ 0{\cdot}00300x_1 &+ 0{\cdot}00600x_2 \leq 100 \\ 0{\cdot}00440x_1 & \qquad\qquad\quad \leq 100 \\ & \quad 0{\cdot}00607x_2 \leq 100 \end{aligned}$$

$$x_1, x_2 \geq 0$$

$$v_1 = \$625 - \frac{x_1}{60}$$

$$v_2 = \$250$$

In practice the objective functions will commonly be given as linear functions, but in general these will be found to be themselves only linear approximations of the true (non-linear) functions which govern the marketing environment of most enterprises outside the short run. Baumol and Bushnell (7) have demonstrated that the discrepancies between solutions based on linear approximations and the true solutions may be greater on average than that between the real solution and *any* random solution. This is the reason which justifies the use of the heuristic approximation of the true 'simplex' solution, notwithstanding the fact that any discrepancy between the heuristic solution and the simplex solution must result in a loss of contribution; the simplex solution itself does not give the true optimal solution either, where the problem is really non-linear.

It will be observed that nevertheless linear approximations are usually to be preferred to the true non-linear problems; this is because the algorithms devised for non-linear solutions are even more prodigal in their use of computational capacity. If one could establish some reasonable criteria for ranking the products in order of attractiveness, it would be possible to devise an heuristic procedure which could approximate the solution of non-linear problems also. The trouble is that one could not be certain of the degree of attractiveness at the outset, since the price obtainable depends upon the quantities sold, which is the solution being sought. A simple example of this technique (applied to the problem set out above) would be possible if a rough guess was made of the relative attractiveness before continuing the heuristic programming technique already described, and the procedure concluded with a check to see whether any substitution between the successful products and the most attractive unsuccessful products would produce an improvement in the result.[14]

All the discussion in this chapter has implied that the data making up the problem are always deterministic; in practice they are always probabilistic. Nevertheless the techniques devised to handle this difficulty, such as parametric programming, or chance-constrained programming (e.g. 31), are not much used in the field; this is mainly because the techniques would exacerbate an already extreme overloading of the computational facilities. However the problems of reliability and probability will be discussed at length in a later chapter, when it will be seen that this neglect of the uncertainties may have a better defence than mere expediency of calculation.

3.8 NOTES

1. The contribution is usually the net sales price of a product, less its variable selling and distribution costs (which leaves its 'net sales income'), less its variable normal cost. As has been observed in Chapter 1, the true economic profit is calculated by discounting the cash flow to present value; it is desirable that if these contributions are used in an objective function for programming purposes, account should be taken of the fact that the cash flows arising from the sales and the conversion and raw material costs of the processes can occur at widely separated points in time. If the factor is significant they should have all their elements discounted to a common base.

Secondly, it is common practice to give different classes of customer different trade discounts, and the distribution costs are likely to vary when a product is sold in more than one geographical market. With a certain loss of accuracy, one could use an average of these items; the alternative is to consider each customer/market variant of a single real product as a separate 'product' for the purposes of planning.

2. As has been explained earlier, a plant may have more than one process going through it, so that its 'actual output' will consist of more than one product. On the other hand, a plant will commonly comprise more than one 'cost centre', for each of which a supervisor will be responsible. Here the actual output for each centre will usually be the same in effect, since it will be related to the throughput of the plant of which it forms part.

3. The 'final accounts' for the ascertainment of the true economic profit contain nothing but 'cash flow' data. At the end of Chapter 1, it was pointed out that normal records of debtors, creditors, cash and inventory would probably be maintained for control purposes, and so the 'actual conversion costs' given above may be taken to be the traditional 'accrual' based costs for the period, which could still be ascertained. These variances only explain the difference between the standard (or normal) data and the accrual data. The reconciliation of these figures to the economic profit will be discussed later.

4. Note that we are assuming that each process *has* an identifiable 'capacity'; this implies that each process wholly occupies a single piece of plant. Since continuous flow-line production is quite uncommon, this is not a very likely assumption. The ascertainment of costs and usages in *multi-process* plant is a much more subjective matter than a non-engineer might suppose. Accordingly discussion of this point is deferred until Chapter 5.

5. In fact some 'bounding' devices have been developed which can often enable constraints of this type to be included without increasing the burden of computation (31). This arrangement will also make it easier to handle the not unlikely condition where a single product can be sold at varying prices in different areas.

6. A paper by Latham (70) gives a general example, while a later piece of work by Hartley shows how rather more elaborate problems could be dealt with (53). The latter paper has prompted Jensen to point out that

problems of joint production are especially likely to invoke questions of non-linear objective functions (67). The next section of this chapter is devoted to various methods of overcoming the fact that $10{,}000^2+$ matrices are too large for solution by the ordinary simplex method of linear programming; Jensen's paper reminds one that 100^2 would prove too large for a quadratic programming exercise! It may be worth drawing readers' attention to an old discussion paper by Gambling and Nour (43) which gives an extra-ordinarily long-winded (but reasonably obvious) heuristic method of resolving *linear* joint production problems of any size; it is merely necessary to carry forward inventories of unused *products* as well as capacities to make more of them. An heuristic technique for quadratic programming is discussed in a later section of this chapter.

7. A reference for further arguments on this topic is provided by Hinkle and Kuehn (56).

8. This solution is in fact:
E 2000; F 1800; B 3000; D 7500; A 4000; G 2500; C 4000 and gives a total contribution of £84,602; this is identical with the true linear-programming simplex solution, which was computed with an elapsed time of 1 minute 33 seconds (admittedly on an old KDF 9!). It is probably the 'bounding' package which was responsible for this prodigal use of computer time.

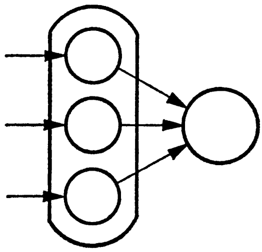
It should be noted that in general the first few iterations of any L.P. usually find the major part of any available contribution. A lot of time can be spent looking for the rest of it!

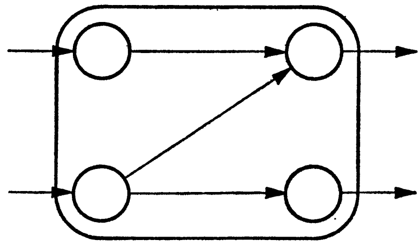
9. Some papers on this phenomenon as used in accounting have been listed in Note 2 of Chapter 1. There we were implying that aggregation tended to limit creativity, and this view was reinforced by some of the references in Note 3 of the same chapter. Clearly this section assumes that a low level of aggregation has been adopted for the general purposes of model building, but that *subsequent* aggregation may be helpful for ease in computation.

10. This phenomenon was referred to when describing the 'Pichler model'; a number of distinct operations can form sub-processes of a single main process, having, perhaps, a number of outputs which nevertheless are jointly produced in invariant proportions. That technique is thus itself a form of aggregating technical data.

11. Briefly, the conversion cost is that of B_3, plus its usages of the conversion costs of its 'ancestors in the chain', B_1 and B_2; the capacity is the maximum output which can be achieved in B_3 because of the bottleneck which is in B_2; the input coefficient is simply t_{AB_3}.

The thesis contained a whole chapter (Chapter 5) on aggregation: it has been omitted because of the author's subsequent enlightenment by Academician Kantorovich and others! These arguments are discussed in the next three paragraphs of the main text. However the original chapter contained at least one other point worthy of a brief mention. One of the Ford Foundation prize winning theses (Tonge (107)) sets out an heuristic procedure for recognising and aggregating patterns in networks. In addition to the 'chain' illustrated in Figures 3.4A and 3.4B, Tonge described the 'set' and the 'Z':





It seems at least reasonable to bear in mind the possibility of more adventurous aggregations: the methods of arriving at their overall costs, capacities and usages are obvious – but long-winded.

12. The calculation of 'the capacity' of a piece of plant is likely to be less than straightforward; obviously one might have to decide between some flat-out technical ideal output, and some lesser figure which is likely to be obtained under reasonable conditions. However this will still not take account of the fact that a (linear) programming solution provides a medium short-term budget and not a specific daily production schedule. This is because the algorithm looks upon the capacity of the plant as a series of reservoirs which can all be flooded simultaneously, and does not take account of the fact that if a product goes through a series of processes it has to go through them consecutively. Moreover the order in which the processes are applied is not often a matter of indifference.

Thus if the capacities are calculated with reference to the operation of the various pieces of plant in isolation, they are liable to be overstated for these reasons, which are known as 'machine interference'. Clearly the capacities should be reduced further to the amounts of output which a reasonably skilled production scheduler could load onto them. It should be pointed out that these amounts are not easy to calculate other than subjectively; production scheduling (and the equivalent problems of flow-line balancing) are problems of combinatorial analysis, which at present have to be solved by rule-of-thumb methods. The thesis by Tonge (op. cit.) was in fact discussing this type of problem; other treatments appear in Eilon (32) and Campbell, Dudek and Smith (17).

13. This problem is posed in Dorfman, Samuelson and Solow (30); the solution is:

$$x_1 = 15{,}000;\ x_2 = 9{,}167;\ \text{Contribution} = \$8{,}116{,}667.$$

14. Thus one might forecast, 'I *think* it might be right to sell about 12,000 x_1's': this immediately establishes

$$v_1 = \$625 - \frac{12{,}000}{60} = \$425$$

Since v_2 is only \$250, the heuristic process would commence with x_1:

(1) Resource	(2) Coefficient	(3) Constraint	(4) 3/2
(1)	0·00400	100	25,000
(2)	0·00300	100	33,333
(3)	0·00444	100	22,522
(4)	0	100	∞

(5) Make and sell		(6) Required resources (5 × 2)	(7) Remaining resources (3 − 6)
22,522 x_1	(1)	90·090	9·910
	(2)	67·567	32·433
	(3)	100,000	0
	(4)	0	100

Taking x_2

(1)	(2)	(3)	(4)
(1)	0·00286	9·910	3,465
(2)	0·00600	32·433	5,405
(3)	0	0	∞
(4)	0·00607	100·00	16,474

(5)		(6)	(7)
3465 x_2	(1)	9·910	0
	(2)	20·790	11·643
	(3)	0	0
	(4)	21·032	78·968

The contribution so far is thus:

$$
\begin{aligned}
x_1 &= 22{,}522 \times (\$625 - \$375) = \$5{,}630{,}500 \\
x_2 &= 3{,}465 \times \$250 = 866{,}250 \\
& \qquad \qquad \qquad \qquad \qquad \overline{\$6{,}496{,}750}
\end{aligned}
$$

However it is apparent that if one made say 5000 less x_1's, this would 'liberate' 20 units of resource (1), and so permit the sale of 5405 units of x_2, in their place (since (2) prevents 6993 being sold!). The contribution becomes

$$
\begin{aligned}
x_1 &= 17{,}522 \times (\$625 - \$292) = \$5{,}834{,}826 \\
x_2 &= 8{,}870 \times \$250 = 2{,}217{,}500 \\
& \qquad \qquad \qquad \qquad \qquad \overline{\$8{,}052{,}326}
\end{aligned}
$$

Obviously in this case we could home in on the true solution (maybe some differential calculus could help!). However given more variables the optimal strategy might prove more elusive. It seems worthwhile mentioning the possibility of heuristic *non*-linear programming, since as Jensen (op. cit.) points out these objective functions are very common in practice, and as Baumol and Bushnell (op. cit.) show they are very dangerous to approximate. It seems unlikely that a practical example could be reduced far enough to permit an algorithmic solution.

4 A Taxonomy of the Information System

4.1 *Quis custodiet ipsos custodes?*

It was suggested at the end of the previous chapter that the practical world of business made comparatively little use of the more highly elaborated techniques of programming. Partly, no doubt, this is because the simpler techniques are sufficiently complex to overload one's computational facilities in a practical example; however a secondary reason is that the data used in the programming are usually probabilistic rather than deterministic, and it is not easy to attach satisfactory measures of probability to real-life data of this type. The latter problem is dealt with quite simply in practice, although the method used is likely to produce a sub-optimal solution; all the data are accepted as though they *were* deterministic, and a result calculated therefrom using the simplest technique available. However this result is not accepted immediately as the plan or budget, but an attempt is then made to improve upon the 'optimal solution' just obtained. The initial calculation throws up the bottlenecks in productive capacity and demand, and the places where reductions in costs would be especially helpful; since the data were not really accepted as being deterministic in the first place, it is felt reasonable now to approach the responsible plant managers and other officers and demand improvements of performance in these critical areas. This approach has led some observers to question the extent to which quantitative data are 'used' in decision making (29). It may be that their use is to provide this starting point for a series of negotiations over what is a feasible plan (see also Chapter 5).

In fact this approach is justifiable under present conditions, as it is a version of the heuristic solution which has already been described. The heuristic programming solution is just a trial shot into a likely area of the feasible strategies; from there the planner takes a 'random walk' to see if he can find some direction(s) where the performance of the system becomes better than at the starting point of the 'walk'. As is usual with heuristic solutions, the improved solution may not in fact be the optimal solution for the expanded system. However the techniques can be defended even in theory, by the argument set out in the previous chapter regarding the dangers of using linear approximations in essentially non-linear situations. The 'optimal' solution of the expanded problem may not be the optimal solution of the true non-linear problem which underlies it. The danger of this method of solution is that it discourages investigation of the possibility either of improving the data, or of making some reliable assessment of their probability. It is possible to draw an uncomfortable analogy. Twenty-five or thirty years ago one would have assumed that fighter pilots would always be vital to air defence. Unfortunately developments in rocketry mean that decisions are called for which have to be made and acted upon more quickly than it takes the best fighter pilot to decide whether to move his right hand or his left. This suggests that so long as everyone else runs his business within a present-day time scale, the current technique suffices. However it also suggests that business organisation might possibly be carried on upon a different and much higher plane.

The succeeding chapters will consider the problems of measuring the probability of accounting data, but first it is necessary to consider the more basic problem of the source of the information produced by the information system. It seems reasonable to say that statements such as a balance sheet or a cash forecast do not have a 'probability' or 'reliability' of their own, but depend on the probabilities and reliabilities of the basic data from which the balance sheet or forecast has been built up. This means that one can distinguish what might be called 'basic statements' which have to be accepted on their own probabilistic merits, from 'complex statements' which are built up from them. Measures of probability attach to material coming from outside the information system only, and other probabilities which may be needed can only be permutations of these basic probabilities; as a result a calculation of probability for a complex statement requires an analysis of the basic data which have comprised it. To do this it is necessary for the information system to be able to provide a model of *itself* as well as of the real system which it is reflecting.

4.2 THE INFORMATIONAL ASSUMPTIONS OF THE MODEL

The model described in the first three chapters of this book has assumed (a) that certain information about the real system was available; it has further implied (b) that these data are costless to gather, retain and manipulate, (c) that they are unambiguous and (d) that they are deterministic. This assumed collection of data contained information:

(i) as to the existence of certain technological processes and their resulting products;

(ii) as to the existence of interconnections between the processes, and the direction of the flow along those connections;

(iii) as to the existence and amounts of standard usages by the processes of the outputs of the ancestor processes to which they are immediately connected for each interconnection listed under (ii);

(iv) as to the existence and amounts of the standard conversion costs per unit of the throughput of each process listed under (i);

(v) as to the existence of certain 'linkages', by which is meant the recognition of some sympathy between the behaviour of various parts of the system which are not directly connected under (ii);

(vi) as to the constraints which may exist on the output of some of the processes given in (i);

(vii) as to the existence of certain markets for the products of the system; and the existence and amounts of some prices in those markets;

(viii) as to the bounds which may exist on the quantities to be absorbed by the market shown in (vii);

(ix) as to the existence of some goals which are being pursued in the management of the real system, and of some criteria by which the 'attractiveness' of different courses of action can be assessed, in so far as they contribute to the achievement of these goals.

In addition the previous chapters have also indirectly implied:

(x) the existence of facilities for collecting, storing and retrieving this information; and

(xi) the existence of a library of procedures for compiling this information into more complex forms.

The succeeding sections of this chapter will make some statements about the further classification of the data needed to carry out the operations described in the first three chapters, as a first step towards examining more closely the way in which it is gathered and built up into the more complex forms required to manage the business. This phraseology clearly implies that the data in the information system are maintained in some intermediate classified form, which differs both from the form in which they were originally gathered, and from the form in which they will be used. In general data exist in three categories:

(a) *Basic statements*. These are the lowest units of information which will concern us, and will comprise (i) 'observations' of a state of nature or the history of some operation over time, and (ii) 'authoritative statements' which are accepted as to what some state of nature or efficiency of operation ought to be. For the most part the technological information whose existence has been assumed so far is in neither of these forms, but rather standard-cost data supposed to be based upon some compromise between the two; thus we can designate a second category as:

(b) *First order complex statements*. These then form the basic modules in the information system, representing the current budgeted prices, usages, capacities and so on. The nature of the compromise necessary to construct these statements will be described in detail in a later section. Finally, there will be the third category:

(c) *Higher order complex statements*. These will comprise an hierarchy of schedules and summaries built up, by the library of procedures, from the basic modules.

Before commencing on the taxonomy itself, or the problems arising from the construction of the basic modules which make up 'the information', it may be useful to point out that this information is merely a subset of some more abstract body of 'total information', whose existence can be conceived but for which no access or means of observation may in fact exist. The operational subset is therefore that information which is (a) relevant to the management and control of this enterprise (b) accessible and observable and (c) believed to be commercially worth collecting and retaining. The exact needs of the organisation for information in the future are probably uncertain, so that the task of the information system is to select, grade and retain a suitable store of basic statements which it is believed will satisfy the future demands.[1] Wisdom unfortunately is not priceless; its value is relative to the probable advantage to be obtained from its use, as against the cost of gathering it, processing it and retaining it. An attempt to achieve absolute accuracy will involve considerable expense while more dubious data will almost always be available much more cheaply; in many cases it will prove preferable to evaluate the latter in some way, rather than demand the former.

Accounting is usually concerned with the collection of information as a by-product of the control system of the enterprise, rather than with *ad hoc* investigation of costs. The reason is that since the data have had to be prepared for control purposes, and probably have to be retained for those purposes also, it costs very little more to use them for informational purposes. However, as will be seen, the data will usually have been prepared for some different and limited original purpose, and so prove less than ideal for more general use. This consideration leads Ijiri (63) to describe accounting statements as 'surrogates' which only indirectly indicate the nature of the underlying reality, which he calls the 'principal'. Thus a decision maker has three choices of data open to him:

(i) he can base his decision on a direct investigation and measurement of the principle;

(ii) he can rely upon the surrogate provided by the general information system, or

(iii) he can simply make a random choice amongst the alternative strategies open to him.

The rational basis for this choice of data must be whether decisions based on *ad hoc* investigation or on the use of surrogate information will produce better results (after deducting the cost of collecting and maintaining them) than a random choice of strategy without recourse to any data at all. Generally the surrogate data will be selected, since its marginal cost is probably negligible while special investigations would be very costly. On the other hand, arguments were adduced in the previous chapter which suggest that, if the non-linearities in the problem are being ignored, the results may sometimes be little improved over those obtained by random choice.

4.3 THE CONTENTS OF THE INFORMATION SYSTEM

Professor Forrester's *Industrial Dynamics* (40) gives a comprehensive analysis of the enterprise as a series of 'flows' which are controlled by a feedback, in turn activated by information about the rates of flow and levels of cash, inventory, order backlog and so on within the system. The flows used in that work are (i) Materials, (ii) Orders, (iii) Cash, (iv) Personnel, (v) Capital investment and of course (vi) Information itself. This book has been concerned exclusively with the way in which the information system reflects the real system, and it seems that little could be gained in such an analysis from consideration of the actual flows within the real system, as opposed to the enterprise's own *understanding* of what those flows may be. Thus all the categories in this taxonomy are 'Information', but it is possible to follow the pattern set by Forrester so that these become:

(i) Information about materials
(ii) Information about orders
(iii) Information about cash
(iv) Information about personnel, and
(v) Information about capital investment.

However it will be convenient to add a further category to the information system, analogous to Forrester's 'Information'; this will be the library of procedures necessary to store, retrieve and compile the data. In effect this is the information about the nodes, arcs and capacities of the information system itself.

The first five categories of information can conveniently be subdivided into seven subcategories, as outlined in Figure 4.1, which is in the form of a matrix, but is not of course one of the matrices contained in the system.[2]

FIGURE 4.1

'Flow'	Input/ Output (1)	Capacity (2)	Prices and rates (3)	Dynamic data (4)	Forecast data (5)	Historical data (6)	'Housekeeping' data (7)
Materials (a)	[$\mathcal{N}$] (extended)	Quotas; inventory policies, etc.	Process conversion costs and overhead costs	Production cycles	(a) Forward contracts (b) Learning curves and growth rates	(a) 'Stack' of superseded data in the other categories (b) Current observations and authoritative statements	(a) Names and addresses (b) Descriptions and locations (c) Serial numbers and codes
Orders (b)	Procedures for bids and orders	Customers and suppliers	Standard prices for sales and purchases	Delays and lags in processing orders			
Cash and funds (c)	Financial accounting	Cash, credit and short-term borrowing policies	Interest rates	Lags in cheque clearances			
Personnel (d)	Procedures for staff training and development	Staff; conditions of service; personnel policy	Standard rates and salaries	Lags in recruitment and training; age and wastage of staff			
Capital investment (e)	Flow charts for capital installations	Plant capacities	Estimates of current replacement costs and disposal values	Times for PERT net-works; age and life expectancy of plant			

The main subdivisions are further divided in the way shown in this figure; the squares in the illustrative matrix can be identified with the various concepts which have been, or will be discussed elsewhere in the book, as follows:

(1) *Input-output data*

(a) 'Material' comprises Assumptions (i–iii) listed above; it is in part the matrix [$\mathcal{N}$] and hence the 'cost accounting' sections of the Chart of Accounts and the Manual of Procedures. This matrix [$\mathcal{N}$] should be considered as extended to include certain nominal 'processes' which reflect the movement of raw materials and other productive resources into the true processes, as well as the compilation of the process conversion costs and overhead costs which are referred to as 'linkages' in Assumption (v). The machinery of these extensions will be discussed in the next chapter.

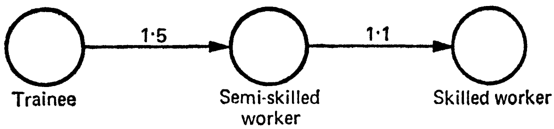
(b) Orders represent the physical flows of enquiries, bids and orders through the enterprise, for both sales and purchases. This data will form the matrix of a graph, similar to [$\mathcal{N}$] itself; in general it will be a series of simple chains, whose principal significance will lie in the extent to which the arcs carry coefficients which are greater than unity. This will indicate the expected incidence of unsuccessful bids, lost orders and cancellations. This sub-section will also designate the saleable products and product groups and the sales areas, etc., for which separate market data are maintained. This is covered by Assumption (vii).

(c) 'Cash and funds' will comprise the 'financial' sections of the Chart of Accounts and the Manual of Procedures. The result is again a matrix for the graph illustrating the flows of funds in the enterprise down to the creation of inventories and funds of the various productive resources needed in the productive processes; their subsequent disposal is carried on through [$\mathcal{N}$] itself.[3]

(d) 'Personnel' will also comprise simple chain graphs, showing the progression of the training and development of personnel (see Figure 4.2).

Here too there is a wastage during the process which is of significance; where ready-trained personnel can be hired, the situation is similar to that where partly finished goods can be bought in for a process as an alternative to processing it internally. Where this facility does not exist, the chain is likely to be of significance when combined with the 'dynamic' data, when it will show the time lag which must elapse before additional labour of the requisite class can become available.

FIGURE 4.2



(e) 'Capital investment' will be a flow chart illustrating the supply, delivery and commissioning of new equipment. In a simple case, this would comprise a single arc and node beyond the quotation and order graph described in (b) above, but clearly more complex installations involving several contractors or the enterprise's own labour would require a more elaborate network to be prepared, which when evaluated with the relevant 'dynamic' data, will become a typical PERT/CPM network for the operation.

(2) *Capacity data*
(a) 'Materials capacity' might include the overall capacities of the 'processes', as opposed to those of the plant itself; clearly this data would need to be derived from the plant capacities, and so represent a higher order of complex statement than the latter. Other constraints may limit the flow of materials through the works, either directly or indirectly. These could include quota agreements, limitations on physical transport between processes, and the inventory policies of the enterprise. This sub-section covers Assumption (vi).

(b) 'Orders' would include, for this sub-section, lists of customers and suppliers, together with estimates of their potential consumption or output and the extent of competitive activity amongst them. One could also include potential suppliers and customers here, so that the data would also contain, on the sales side, the sales forecast figures. These data are referred to under Assumption (viii).

(c) 'Cash' capacity will comprise the cash holding, credit and short-term borrowing policies of the enterprise, together with any limited facilities which may have been negotiated in this area.

(d) 'Personnel' capacity will contain lists of staff and employees classified by their various grades, skills or locations; details of the law, local and national agreements and company policy regarding conditions and hours of work; also information on the current state of the labour market.

(e) 'Capital investment' capacity will comprise the capacity of the plant; this will probably be extended to include ancillary information linking the machine tools themselves to other indirect constraints, such as their power, labour and service requirements.

(3) *Prices and rates*

In general these items will be self-evident, and they are the means by which cost data originate in the system; as will be explained further on, the prices, etc. will be standard prices, in the same way that the inputs in (1) are standard usages, and so they will usually be complex statements of some degree.

(a) The 'materials' data here will be the standard conversion costs per unit of throughput in the process, the subject of Assumption (iv). These will be complex statements, based upon the prices and rates retained elsewhere in this sub-section and the data referred to under (1) (a) above.

(b) 'Orders' will here list all standard rates and prices; these relate only to exogenous resources which originate outside the system. They represent the balance of the data assumed under (vii).

(c) and (d) 'Cash' and 'Personnel' will record standard interest rates and wage-rates and salaries, respectively, in this sub-section.

(e) The price-data for 'Capital investment' will comprise current estimates of replacement cost and disposal value.

(4) *'Dynamic' data*

These are self-explanatory in each case. The data appearing in sub-section (1) refer only to input-output usages, and take no account of the passage of time while the process is in operation.

The substitution of standard operating cycle times will convert all the graphs described in sub-section (1) into the appropriate PERT/CPM networks. To the extent that (d) 'Personnel' and (e) 'Capital investment' describe wasting resources, data about the ages of men and equipment might also be included here.

(5) *Forecast data*

There is very little 'information' available about the future, except to the extent that it relates (a) to forward contracts which are likely to be fulfilled, (b) for the rest, forecasts must be extrapolations of past and current experience, taking account of expected growth rates, learning curves and the like;[4] the data of this type, used to forecast the future states of the current data carried in all the sub-sections (1) through (4), will be carried here. These data will be the result of a feedback which compares current performance with expected performance, which will need to be explored in detail in a later chapter.

(6) *Historical data*

The data carried in sub-sections (1) through to (4) will be a single, currently-expected 'standard' in each case. The information system is also required to carry substantial quantities of data under the various 'flow' headings from which these standards can be continuously updated. In part these will be (a), as it were, a 'library stack' of superseded standard data, which it will be seen may form an element in the calculation of new standards. There will also be new, untreated data; these are the basic statements described earlier in this chapter and comprise both current observations and currently-received authoritative statements.

(7) *'Housekeeping' data*

A substantial part of the contents of the information system will be material which adds nothing to an understanding of the real system's condition, prospects or history. These are descriptive labels of various sorts:

(a) There are those which enable the system to establish contact with exogenous agencies, which comprise names, addresses and telephone numbers.

(b) Endogenous agencies will also require descriptions and locations.

(c) Especially where automatic data processing of some sort is in use, it will often be convenient to be able to refer to both exogenous and endogenous agencies by means of a numerical code number; this sub-section will also contain the directory material necessary for this.

These labels are necessary to enable the library of procedures to store and retrieve data in the data bank.

4.4 AN EXCURSION INTO THE BEHAVIOURAL THEORY OF THE FIRM

In the opening paragraphs of this chapter reference was made to a dichotomy between academic analysis and management of practical affairs; neither academics nor managers are to be criticised for the existence of this gap, since the facility to abstract problems from the complex and uncertain circumstances which surround them in practice, permits the academic to develop possible explanations for real behaviour which could not otherwise be illustrated. On the other hand, it is undesirable to suggest that decision making techniques developed in this way can be applied immediately to a real problem.[5]

At various places in the earlier chapters the objectives of the organisation have been stated in terms such as 'the maximisation of the present value of the enterprise'. It is now necessary to expand somewhat the reservations with which these statements were surrounded; it has been suggested by Cyert and March (23) amongst others that this objective is rather abstracted from reality. Instead it is proposed that the basic objective of any organisation is much more primitive – in effect simply to survive and (if possible) to grow. Moreover the organisation can only act through engaging the services of external, human agents, who probably can only be motivated to advance the objectives of the organisation in terms of their own satisfactions, whatever they may be. This has brought about the development of the 'behavioural theory of the firm', and it is necessary to consider something of this, in order to explain something about the probabilistic nature of the data which are to be found in an information system.

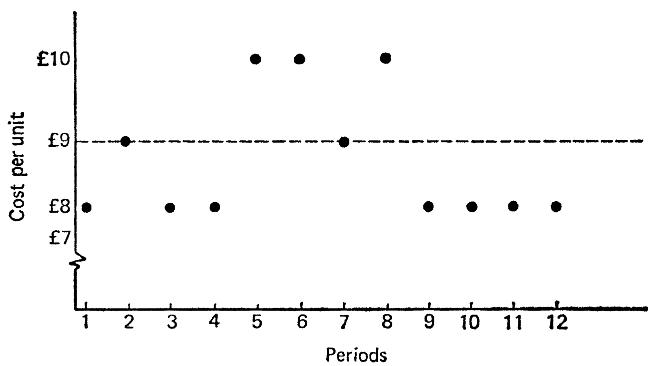
The aspect of this behavioural theory, which particularly relates to accounting and information, is the concept of 'organisational slack'. It is obvious that a cost centre could be operated at an extreme state of efficiency to give rise to a minimal cost per unit of output or service. This limit, which might be called the 'rack cost' (or perhaps the 'salt-mine cost'!) of its operation, would be achieved when everybody was working so hard, wasting so little and spending so little on personnel comforts like lighting and heating, that if they tried any harder they could not support life. One might also consider a 'maximum cost' for its operation, which was so wasteful that the organisation as a whole could only survive if all the other cost centres functioned at *their* rack costs and the shareholders' dividends and growth rates were cut to a similar survival level. Clearly the cost centres will not be allowed to operate at either extreme, but rather at some intermediate level between them. It will be necessary to discuss the machinery by which this intermediate 'standard' or 'normal' cost will be negotiated a little later, but at present it is sufficient to establish the point that costs are not entities which are established by natural law. Rather they are *negotiated* much in the way prices are negotiated between trading concerns; in short every cost has much in common with the 'transfer price' which has to be negotiated between the divisions of a vertically divisionalised firm.[6]

As a result every normal cost will contain a slack element, representing the surplus which has been allowed over the rack cost. This slack serves at least two functions. (a) It permits smoother personal relations within the cost centre, to the extent that the supervisor is not compelled to take adverse notice of every infringement of economy, and the worker may trade off more congenial discipline and surroundings against specific demands for improved conditions and rewards. (b) Another function is served by the opportunity it gives a supervisor to create 'secret reserves' within the cost centres. These reserves, it is argued, are an essential prerequisite for the smooth operation of any hierarchical system. In such a system a supervisor will best serve his interests by avoiding both extremes of performance; this is because the enterprise is being directed in accordance with some policy, or grand strategy, laid down by senior management. The local supervisor is given full authority to devise the tactics necessary to carry out the overall policy, but at least in theory it is as undesirable for him to exceed what is asked of him as it is to fall short, since the result can only be to disturb an otherwise optimal plan.

In practice of course no one is likely to be exactly on plan, so that overfulfilments in one area are needed to make good shortfalls elsewhere. However according to this line of analysis a supervisor will find that his enhanced standing as a result of supernormal performance is not so great as his loss as a result of falling short; moreover he will be expected to continue to achieve these improved results in the future. In a scientific experiment it is the researcher who makes the observations and measurements necessary for the research; this can be contrasted with the situation in an hierarchically organised system, where it is the subordinate who is required to report to his superiors on his own performance and those for whom he is responsible. It is suggested that in these circumstances it is inevitable that some 'bias' will enter into such reports, whereby both underperformances and overperformances will be minimised. People are clearly happier to hear (and say), 'We're right on target,' rather than the more precise statement to the effect that 'We are no more than reasonably off target!' At the outset, it is to be emphasised that this under-reporting is by no means always to be seen as an attempt to conceal the truth from a superior officer. The performance of machines, people and organisms are rarely at a steady rate, but show certain oscillations about some central trend. Sometimes these oscillations will be entirely random, but other activities will show cycles of above average and below average performance; this fact is readily understood where the cycle is completed annually, but where the cycle exceeds a year's duration the traditional accounts will disclose 'good' and 'bad' years. Consider this artificial example; the cost per unit in a centre has varied between £8 and £10 for a long period in such a fashion as to produce an average cost of £9. During the last four periods, the observed cost has been consistent at £8 (see Figure 4.3).

In themselves these results provide no answer to the question, 'Has the cost now settled down at £8 for good?' or alternatively, 'Can we now expect a run of £10s to counterbalance the run of £8s and confirm the average of £9?' Only someone with a real knowledge of the centre can even form an opinion about this; if he has reason to know that the centre is now functioning more efficiently, he will accept the former hypothesis, but otherwise he will prefer the latter. Accounting is essentially a matter of 'smoothing' observed results and attempting to discover the central trend of the costs, and it will be necessary to consider the various techniques of smoothing which might be adopted, at a later stage in the argument. However at the moment it is sufficient to observe that if periods 9–12 constitute a trading year, the average cost (in traditional terms) for the year would be £8.

FIGURE 4.3



If the trend has changed, it will correct if the management requires the centre to continue to operate at £8 per unit in future. On the other hand, if the trend is still £9, and moreover the statistical probability requires a run of £10s, the centre and its supervisor will have troubles if the normal cost is reduced to £8. Of course he could try to explain his belief that the trend was really still £9, but he has a means at hand to deal with this problem that is much more likely to hold the accepted normal cost at £9. He can requisition raw materials, etc. at a rate of £9 per unit during periods 9–12, and so build up an undisclosed surplus inventory of £1 for every unit produced during the period. In the following periods, if his guess is right, he will be expending £10 per unit, against which he can only requisition £9; however this shortfall is made good out of the undisclosed surplus. Many more such devices are in use, such as advancing or postponing expenditure between accounting periods, varying the treatment of indeterminate 'capital' and 'non-capital' expenditure, but this text is not concerned with the techniques of creating secret reserves. The point is that the executives who create them probably see no fraudulent element in the procedure at all. They take nothing for themselves, and in a sense they are just creating reservoirs to take care of the statistical 'white noise' in the system.[7] It might be noted that 'economic accounting' always smooths profit figures and cannot show such a thing as a good year or a bad year, but only a good project or a bad project; there seems to be some theoretical justification for the practice.

It might be thought that reserves of this sort must correct themselves over a longer period, since it is usually the inventories, repayments and accruals which are adjusted rather than the transactions themselves, and the closing inventories, etc. of one period form the opening entries of the next. This is not always true; from time to time some disaster will occur, such as a fire, a burglary or a labour dispute. These are periods when no one expects a typical performance from the centre, and so inconvenient balances can be carried forward from period to period and then allowed to fall into the costs at such times. The practice of creating these reserves is dangerous, since there is no dividing line between a sincere conviction that the reserve simply covers noise, and an attempt to cover up shortcomings in the belief that some disaster (which is sometimes even induced deliberately) will occur which will enable the item to be lost sight of.[8]

Since the 'observations' are usually controlled by the responsible executives, one could hardly measure the amount of organisational slack which exists due to biased reporting; as has been said, it is not really useful in practice to distinguish between the real system and the information system's model of what it understands that system to be. It is probable that secret reserves will form a fair proportion of any discrepancy between the two. However it is useful to recognise the probability of its existence, since this will aid the analysis of the probability, or rather the reliability, of the data in the information system. On the other hand, it might be easier to estimate the extent of the organisational slack which is permitted for amenity purposes, as will be discussed in Section 4.6.

4.5 SMOOTHING AND FORECASTING

The dilemma posed by Figure 4.3 is a central difficulty of any forecasting operation. It has been pointed out that one is looking for two totally incompatible facilities in an information system: it should be highly sensitive to changes in trend, but highly insensitive to statistical noise (13). The forecaster is confronted with a current normal trend which has been accepted on the basis of past experience, plus a new piece of data, which may differ from this trend. As was suggested in the previous section, it is possible that knowledge of the circumstances under which the system was operating might provide an indication of the probability of a change, but this is likely to be a subjective estimate. As a result it will usually seem best to arrive at a new expected value by combining the old trend with the new observation. This has the effect of backing both of the alternatives as between noise and change: the new value can however be made more or less sensitive to the effect of the new data. This is brought about by adjusting the weights attached to the old reading and the new in the smoothing calculation.[9]

It is important to appreciate that smoothing of data is in any case an inherently subjective operation. A number of alternative methods are used to achieve the result. An obvious choice might be to take an average of all the observations that have ever been taken; this could be made more sensitive to change by taking a moving average of only the last n observations. In practice it might seem to be impracticable to retain n individual pieces of data simply for this purpose, so a more usual method is the one implied above; there, some average of the new and the old *average* was used instead. This is known as 'exponential smoothing', and the weight given to the new observation is conventionally designated α and that for the old average β, where $\alpha = 1 - \beta$.

The real nature of the operation is to fit a line through the series of past observations, including the current observation, which can be extrapolated into future time periods, as a forecast of the future expected behaviour of the series. Again, this is a complex subject which has been well explored by operational research workers and systems engineers; it is possible to differentiate between the suitability of alternative methods of averaging and weighting, or between the techniques of extrapolation to be used once the 'line' has been established.[10] The purpose to the latter part of the present book is to construct a model of the information system; it is sufficient to know that the normal data on external phenomena for the coming period is likely to contain, perhaps amongst other elements, a current observation with the weight α and the current normal data with a weight of β, plus some 'forecasting' function δ. For our purposes we assume that δ is simply some decimal fraction which is used to multiply the sum of the other items; in practice it might be the function of a learning curve (see Note 4 of this chapter). The source of α, β and δ will initially be considered to be an exogenous 'authoritative statement' which is just accepted into the system, although reflection will suggest that the weights selected will themselves be the subject of a 'feedback mechanism' which will take account of the error between the forecasts arrived at in this fashion and the actual observations for the various periods. Initially this assumption will be made about the current normal costs and other data, although these will clearly have been produced within the system by a similar operation in the previous time period. This is justified, because the model of the information system which is being described in the first instance is purely static, and considers data produced endogenously in a previous period of time to be exogenous from the point of view of the current period. At a later stage this assumption will be relaxed to produce first a semi-static and then a dynamic model, which will then be able to take account of the feedback within the system.

It should be observed that smoothing has been discussed exclusively in connection (a) with the updating of normal costs at discrete time intervals and (b) with reporting to an hierarchy, also at discrete intervals. Of course the example shown in Figure 4.3 shows average costs in a series of 'periods'; it could equally well be a number of actual observations taken within a single period. In that case, although the results would certainly be smoothed at the end of the period, the question would also arise as to whether the process was 'under control' *during* the period (see Section 7.1).

4.6 THE NEGOTIATION OF THE BUDGET

Turning to internal cost data, it has been suggested that instead of the smoothing process another element enters into the final settlement of the normal costs for the coming budget period. The observed costs will have all included (varying?) elements of discernible organisational slack, so that some part of the variances between the normal cost and the actual cost may represent a failure by the supervisor to hold the slack element to the agreed rates, rather than some inevitable pressure from the technology or environment of the enterprise. This means that each budget will be the occasion of a further round of negotiations between the supervisors and plant managers with the central administration of the organisation. An analogy can be drawn between this procedure and the negotiations leading to the settlement of a transfer price within a divisionalised enterprise; this in turn has analogies with the negotiations between a trade union and an employers' federation, in that for practical purposes there is only one buyer and one seller. This situation is known as a bilateral monopoly, and it has been well explored by economists for many years (22), both in general terms and with particular reference to the divisionalised (58) and the collectivised (50) firm.

Economic price theory is concerned with the provision of rational explanations of the formation of prices. The peculiar feature of bilateral monopoly is that normal techniques of economic analysis cannot find any one price towards which the bargainers will be forced; instead analysis suggests that an indeterminate price will be found within a certain range of prices. Of course prices are eventually agreed, but it would seem that the precise point in the range is only determined by influences which are not readily quantifiable, such as the personalities of the negotiators, the existence of some *status quo ante* in the price, the time allowed for the negotiations, or the availability of alternative or substitutable resources (41). Since the purpose of this section of the book is to analyse the processes of the information system so as to be able to assess the reliability of the data produced by it, it will be necessary to look into these negotiations in some detail. The feedback from the observed results to the calculation of current normal cost is obviously much complicated by the existence of this indeterminacy.

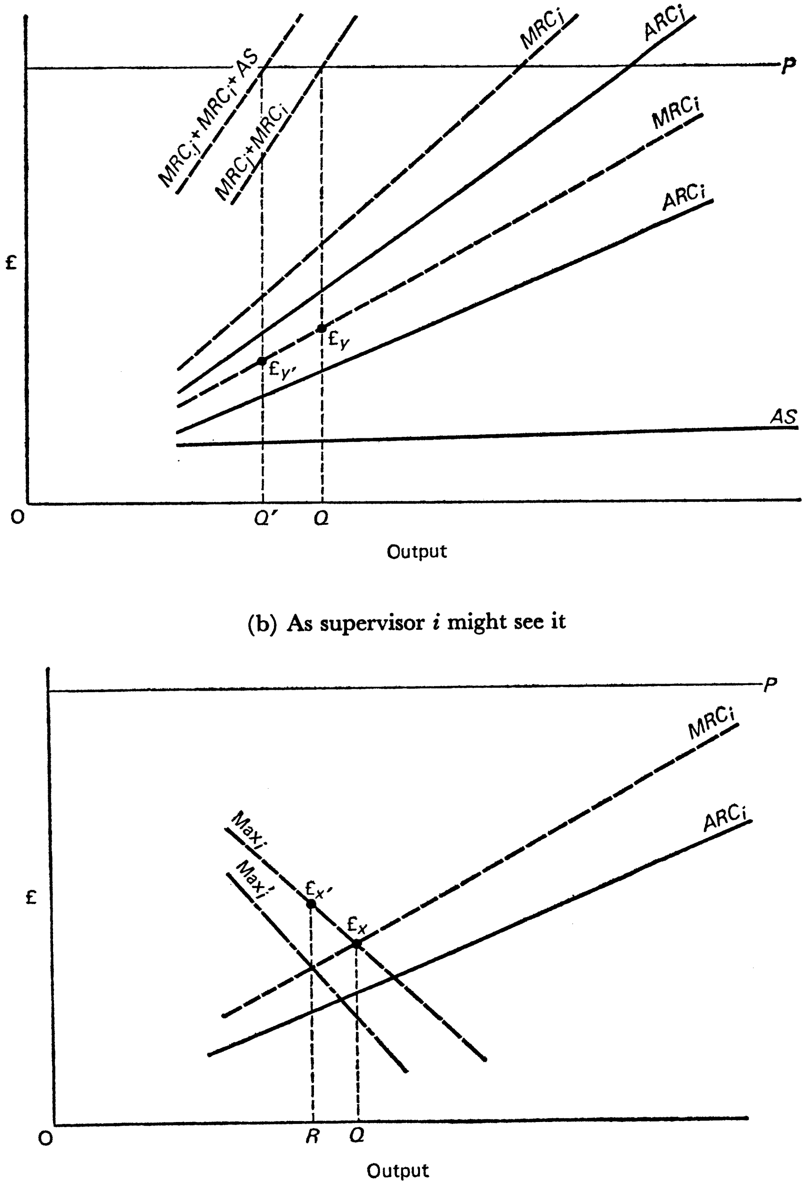
Initially it is assumed that both parties to the negotiation have perfect knowledge of the income available to the enterprise and the costs relating to the various centres. It should be noted that the cost centres do not negotiate directly with each other in the budget settlement; each centre deals with the central administration, which has to strike the best balance it can between the conflicting interests of the centres (with regard to organisational slack), the workers (as to wage rates) and the shareholders (as to dividends and growth). Accordingly one must suppose that the bargainers will not only know the observed cost (which is no more than a guide to the ability of the supervisor to fulfil the bargain struck at the last budget), but also the rack cost of the centre under discussion *and* those of the other centres and interests in the enterprise. One might wonder whether in reality the rack costs are discoverable. Presumably work-study exercises ought to discover them but it is not unknown for workers (and others) to fail to co-operate fully with work-study teams and attempt to suggest that the normal expected performance lies some way above its real point. Nevertheless it seems reasonable to suppose that both a supervisor and a budget controller may have an informed opinion of what the rack cost might be. It seems less probable that any one supervisor will have a very clear impression of what the rack costs may be elsewhere in the system, and so he may not have so clear a view of his own maximum cost. This lack of full knowledge will not upset the analysis; it simply makes the bargain even more indeterminate, since one party will not then have a clear understanding of the probable upper bound of the cost range.

Given this assumption of perfect knowledge, some interesting problems arise in this sort of bargaining. Adapting Hirschleifer's (58) example of divisional transfer-price negotiations to the budget negotiations within a single division, Figure 4.4 would be a graphical representation of the situation.

This analysis assumes that the enterprise is engaged in making a single product,[11] for which an unlimited demand exists at a given price; also the marginal[12] rack costs are rising, both in cost centre *i* and (at least overall) in the other costs centres of the enterprise. As usual the analysis requires us to consider both

FIGURE 4.4 Single product cost-negotiation

(a) As the firm sees it



(b) As supervisor *i* might see it

the marginal costs (shown as broken lines), the average costs (solid lines), and also the irregularly broken lines, which represent the marginal cost, or revenue, *to the other party*, acting on the assumption that the first party will insist on recovering at least its marginal cost for the last item on *all* the items transferred, so that what is marginal to the first party becomes effectively the average figure to the second party. This is the main point of this exercise; it is possible to assume that each supervisor is acting as if he were an independent entrepreneur, without thought to the 'common good', and hence that if he were to be forced to accept less than the marginal cost of making an item, he would not make it. It may be that this assumption is not entirely justified.

The significance of the notation is as follows:

P	The constant price of the single product.
MAX_i	This is $P - MRC_j$, where MRC_j is the marginal rack costs of the other centres. This assumes that the administrators and other supervisors cannot be driven below marginal rack cost, and so that they will insist on retaining at least the amount of slack represented by $MRC_j - ARC_j$.
MAX'	Thus it becomes possible that MAX_i, although based on marginal costs, will become the average maximum cost obtainable by centre i. In such a case, it would then be possible to construct this further line which represents the *marginal* maximum cost per unit.
MRC_i, ARC_i, MRC_j, ARC_j	These are the respective marginal and average exogenous rack costs.
AS	The total allowable organisational slack per unit.

The reason for assuming that the supervisors might not be persuaded to accept a cost below marginal rack cost is simply that they would always be better off, or no worse off, if they then produced some smaller amount. It might be thought that internal works discipline (which might not apply to Hirschleifer's divisional managers) could ensure that production reached any required level within the capacity of the centre; against this it might be remarked that the supervisor also tells the works manager what the capacity of his plant is *supposed to be* in the first instance! If this argument is accepted, it would seem that supervisor i would like to produce the quantity R and charge £X' for it because at an output of R, MAX'_i is equal to MRC_i.

This is where the present analysis departs from that used by Hirschleifer; the output at which $MRC_i + MRC_j = P$ is at the point Q, where MRC_i is also the same as MAX_i. This would indicate that MAX'_i was a 'fair' division of the slack, and any negotiations here would be towards £X or £Y, which would be the average figure at which one party would enjoy no slack at all. Of course the rack costs are notional costs and not the real costs which will be incurred by the operation. In the divisionalised firm, the transfer price determines the division of the total profit between the divisions, but here there is *no* such profit to be divided, since the shareholders' subsistence return has been included as a form of rack cost. Thus if Q is the point at which the total rack costs equal P, and we assume that real marginal costs, which are $MRC_j + MRC_i + AS$, cannot be forced back to these rack costs, it is clear that the maximum production will be where P is equal to the total real marginal costs, namely Q'; by definition, this quantity will be less than Q and so introduce £Y' as another 'possible' lower bound for this cost.

It has been pointed out by Gordon and others that this analysis can only apply where the marginal costs are rising, so they are at all times greater than the average cost. When costs are falling, the marginal cost will be below the average cost, so the supervisor will have an incentive to manufacture as much as possible. This is true, but it is only part of a larger argument as to whether cost curves are U-shaped or L-shaped.[13] If an L-shaped curve is assumed together with a constant price, it is apparent that an infinite output would be the optimum output. Of course in such circumstances the output would be limited by plant capacity or an upper bound upon the demand for the product; this presupposes that the purchase of additional capacity would produce a prohibitive increase in cost, or a reduction in price, or both. In short, the L-shaped curve is the short-term picture while the U-shaped curve reflects the longer-term position.

However the detail of the negotiation machinery need not concern us at the present. It suffices that the input into the negotiation, which might be called the monitoring process, will be the expected cost, which is to be adjusted with reference to the smoothed expected income, and the relative rack costs of the centre in question and the other centres.

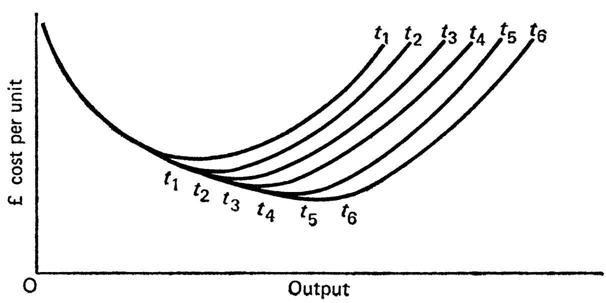
4.7 THE BEHAVIOUR OF NEGOTIATED TRANSFER COSTS

It can be argued that if one accepts the validity of the suggestion that all normal costs are negotiated in this way, it may be a more complex matter than might have been otherwise believed to arrive at convincing measures of their reliability; however this will be discussed in a later chapter. Of more immediate moment is the proposition that if the argument is right, there is an element of organisational slack in every normal cost. Quite apart from the influence of probabilistic error, this slack would go towards an explanation of the phenomenon noted in the opening paragraph of this chapter, namely that management commonly expect to improve upon 'optimal' strategies. Every cost comprises a 'soft' bulk of organisational slack surmounting the irreducible base of the rack cost, and so any one cost can be reduced to its rack element provided that the total amount of slack available to the responsible workers and their supervisor is not greatly reduced. It is not too fanciful an image to speak of the costs 'fitting in' to the interstices of the feasible capacities of the system. A number of important consequences seem to flow from this:

(a) It has been suggested that at present the negotiation of the normal costs implies a sort of pre-optimisation. More realism could be brought into these negotiations by using the rack costs, or more precisely the rack usages, in the linear programming operation, before starting the negotiations. This would disclose the unclearable bottlenecks in the system, and the slack could then be permitted only in the unconstrained areas.

(b) Normal costs are negotiated for a limited period; both parties expect to renegotiate before the next budget period. In general this period will not exceed twelve months. However many firms also attempt to prepare long-range forecasts, and it has been shown how the concept of economic accounting depends upon the existence of fairly reliable forecasts covering the remaining lives of the various projects. This makes it necessary to consider how normal costs can be expected to change, if at all, with the passing of time. It is commonly said that over time all costs become responsive to changes in production; by this is implied the relaxation of two of the assumptions of the simple production function described in Section 1.2, in so far as changes are now possible in the 'fixed'[14] inputs and the selection of the technology. For example in the longer run it is possible to close down plant completely, without having to consider the fixed establishment costs which are inescapable in the short run.

FIGURE 4.5



Wiles (15) has observed that the empirical evidence usually shows L-shaped rather than U-shaped cost curves; earlier in this section it was suggested that the L-shaped curve was appropriate to the short run, while the U-shaped curve might be a reasonable representation of the long-term conditions. A possible explanation of the apparent continuation of the L-shaped curve over a long period of time during which output increases would be that technological change has the effect of continually flattening the base of the U; production of course never goes beyond that point (see Figure 4.5). The forecast of costs for use in long-term planning exercises would need to take account of probable changes of this sort over time. It is apparent that in any case only the future behaviour of the rack costs can be forecast, since the slack will accommodate itself to whatever constraints are binding upon the system at the time.

(c) When one talks of 'responsive' and 'unresponsive' costs, it is necessary to point out that one does not often find an actual account heading whose contents could be said to behave in this fashion. Instead most costs will be of a semi-variable nature even in the short run, so that the terms 'responsive element' and 'unresponsive element' only can be applied respectively to the origin and gradient of the regression line which is being used as the linear approximation of this cost. It could not even be assumed that this unresponsive element must represent the irreducible element in the cost, since, as was demonstrated in the previous chapter, a linear approximation can consist of several segments; it cannot be assumed that the earlier segments would have the same point of origin, at least in those cases where there is no empirical experience of production at the lower levels. One might also ask whether organisational slack could also be divided into responsive and unresponsive elements. Probably it is only the rack costs which might be expected to reveal a meaningful functional relationship as output changes, since the slack element of any one cost can vary quite arbitrarily, usually in response to changes in other parts of the system.

It is suggested that the concept of 'rack costs' and 'organisational slack' is a useful contribution to cost analysis, which explains a number of difficulties experienced in practice. The difficulty is that while the total costs are observable, the rack cost of an operation is not often physically achieved. However it may be that the concept is not unmeasurable; and in some cases it is in fact measured. Manuals on time-study practice (e.g 4) rate normal performance either on the Bedeaux 60/80 scale or the B.S.I. 75/100 scale. The rating of 120 (or 150) is described in terms such as 'Exceptionally fast; requires intense effort and concentration and is unlikely to be kept up for long periods'. Perhaps this measure could be put forward as the rack performance of labour,[15] while 'theoretical chemical yields' or engineering equivalents would be the rack consumption of materials.

4.8 NOTES

1. Sorter describes this as the 'events' approach to Accounting Theory (103). 'Instead of producing input values for unknown and perhaps unknowable decision models directly, accounting provides information about

relevant economic events that allows users to generate their own input values for their own individual decision models.' This approach was also commended by Wheeler (113) at least in so far as completely comprehensive decision models have not been developed.

It must be admitted that this chapter comes down quite heavily in favour of at least the possibility that a 'data bank' *might* exist. References to contrary opinions are quoted in Notes 2 and 3 of Chapter 1. The author holds this view because he believes that every man (and hence every accounting system) has a *Weltanschauung* in terms of which he makes his decisions; hence the decisions follow from the taxonomy and not vice versa. The matter is very fully discussed in *Societal Accounting* (47).

2. The reader is again referred to *Societal Accounting* (op. cit.), especially the two illustrations on pp. 119–20. These attempt to show how Figure 4.1 of this book fits into the cultural 'mazeway', which is the vehicle for the *Weltanschauung* referred to in Note 1 of this chapter. It will be seen from these illustrations that the mazeway is highly selective over what it will admit as data: these cultural filters were deliberately not discussed in this book. Nevertheless such considerations must underlie most of the material to be discussed in the rest of the book. An Anglo-Saxon cultural ambience must be allowed for throughout!

3. It will be observed that this distinction between the technical and financial flows of funds reflects the supposedly old-fashioned approach to cost accounting, through a Cost-ledger Control Account (e.g. 114), rather than the integrated cost accounting systems more commonly shown in texts today (e.g. 62). In practice this division can be found in the accounting procedures of most large enterprises; the control account itself is rarely met with in the books, but quite distinct groups of people will commonly be responsible for the calculations required for the two parts of the operation. Incidentally this division of functions is also to be met with in a more uncompromising form in the Soviet Union, where the Chief Accountant and his or her staff deal with the *bukhgalterskii uchet*, which covers only historical, financial accounting, while the management accounting operation is carried out (as *statisticheskii* and *operativno-statisticheskii uchet*) within a separate 'production planning department', under a Chief Engineer or Chief Economist! (105).

4. 'Learning curves' are a topic which seems to hold out promise of considerable importance in accounting; they have not in fact developed much beyond their original war-time (American) state. The blockage may be due to Anglo-American accountants' enthusiasm for L-shaped cost curves and historiography, both of which are ill-suited to dynamic, systems-oriented development. A recent treatment of the learning curve phenomenon is provided by Morse (84).

5. The thesis of this book is in effect that a good deal of the input data called for by the various decision-making algorithms are available only in a rather imperfect form. The real task ahead of accounting research may be to remedy this. If so, the contribution of accounting to decision making is at an altogether earlier stage than that of economics or even operational research. Of course abstract arguments always can be carried on if one cares

to use 'abstract' data; it is apparent that there *are* no accounting problems at all in a perfectly abstracted world.

6. A paper by Onsi (86) gives empirical evidence for the existence of this slack element. Unfortunately for our immediate purpose, the evidence is drawn from a subjective questionnaire to managers and so does not provide any measurement of the amounts involved.

7. The author once checked the inventories of a substantial branch enterprise and found some gross manipulation over a long period of years. When questioned the local director said, 'Ah, so you noticed that – I was wondering when someone would! You see the way I do the accounts is this; I get them completed without the inventories, and then I can see what the inventory values ought to be. If I didn't do that the profits down here would fluctuate and the main Board would get worried quite unnecessarily.' When his attention was drawn to the provision of the Companies Act regarding secret reserves he replied, 'But that's how everybody does it, you know!' In the event it proved impossible to convince this director, who was a loyal, able and patently sincere man, that there was anything wrong in this practice; indeed in the end it was necessary to override his accounts. These events took place in the mid 1950s; the man concerned was not an accountant or lawyer and had never heard of the case, *Rex v. Kylsant and Morland*, which had ruled against such practices in 1932, but during the ensuing discussions he faithfully reproduced most of the arguments for the defence in that trial (92). Some further examples are also listed in a recent paper by Seed (99). A rather more specialised type of smoothing is discussed by Barefield and Comiskey (6).

8. M. Schiff and A. Y. Lewin (98) also confirm the existence and perhaps the necessity of organisational slack within a budget. However they do not follow the author in suggesting that the costs themselves therefore must be analogous negotiated transfer prices; instead they adopt a more 'Tayloristic' approach and see the task of management as 'forcing divisional management to *rethink* their assumptions'!

9. It can be seen that what one means by the term 'smoothing' depends upon whether one is talking about internally or externally generated data! Internal data will relate either to supposed technological certainties, or to items which are negotiated between the parties concerned: in such cases the 'smoothing' will take respectively the form of creating secret reserves or organisation slack (see also Note 5 to Chapter 5). By contrast external data are likely to be of a 'black box' nature, and the uncertainty surrounding their behaviour will be admitted. Clearly this section deals with the latter type. Of course the secret reserve and the organisational slack will have much in common: they both oil the wheels of the hierarchy. As such they form part of the 'super-bezzle' – a slack element in the National Income itself (see *Societal Accounting* (op. cit.) at Section 6.3).

10. From a dynamic point of view it is unlikely that we would stick to any one system of weighting 'through thick or thin'. (a) If we are satisfied with our method of forecasting we will give great weight to past results either in our extrapolation or as evidence in the parameters of our 'model'. (b) However the deviation between the observed results and our forecasts

may become too large and we might try a new method of forecasting – either giving more weight to recent results or using a totally new model. (c) If neither strategy reduces the deviations overall, we just have to accept that our activity is becoming more chancy! This reminds us again of the paper by Elliott and Uphoff (33) which demonstrated that an econometric model proved superior to simple smoothing as a method of forecasting. Also Rivett makes the point more than once in his book that extrapolation without a model is just the adoption of a very simple-minded model indeed (94).

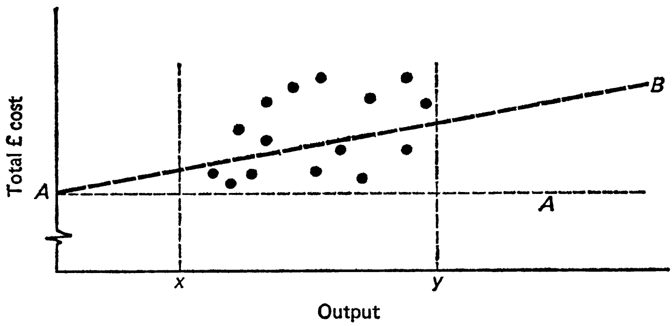
11. The single-product example is used so as to capitalise on Hirschleifer's well-known analysis! However it will usually be necessary to consider a firm comprising a number of multi-product plants, and one in which any one product will pass through a number of individual processes. Clearly this bargaining must still occur, but the upper bound, or the maximum overall cost, will be much less apparent because each plant/cost centre will be contending for a *share* of the available slack, not only against the budget controller, but also against the other plants/cost centres. Also, slack can occur either in the exogenous conversion costs or in the endogenous usages. At the same time the single product plant has either to make its product or go out of business, but the multi-process plant may well be able to function without making some of its feasible products.

Conceptionally the answer must be in some parametric programming exercise which would consider how high each plant could push its cost/usages, while the other plants, etc. were held to their rack costs. Presumably there would be lower bounds set to the activity of each plant so that it could not price itself, or anybody else, out of operation altogether. There seems little point in setting out an example of such a programming problem: it is unlikely that any manager will actually solve them. On the other hand one can see how the approach will aid in understanding the pattern of the intuitive bargaining which will occur over budgets. For example a plant whose output is used in small quantities will be better placed than one which supplies major ingredients or components. Again, one which supplies many other processes will do better than a plant which supplies only a few – in both cases because it would take more to price its products down to its minimum level of activity. A game-theoretic approach might be useful if an n-person theory were better developed!

12. The term 'marginal' is used here in the usual sense attached to it in economic analysis, meaning the additional cost, revenue, etc. which arise from the manufacture or sale of the given item. It is the difference between the total cost of making n items and making $n + 1$ items. When accountants speak of 'marginal costs' they commonly mean the *average* variable costs of production, which is not at all the same thing!

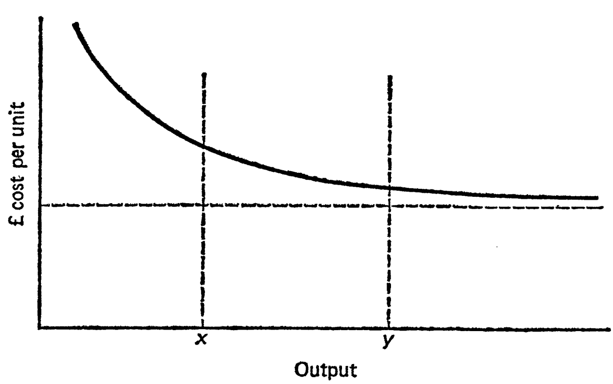
13. The accountant's conventional use of variable normal costs of itself assumes that cost is a linear function of production; that is to say, if the total cost for the various levels of output were to be plotted on a graph they would form a straight line, rising at an angle from some point on the vertical axis. It has been argued that it is unlikely that any costs can behave in this way in reality; empirical evidence suggests that they often seem to do so *over the*

range of outputs used by the management. This apparent behaviour is reflected in the flexible budgeting technique which divides all costs into one element which is linearly responsive to activity and another which is totally unresponsive to it; as a result the average total cost per unit is assumed to fall continuously as production expands. This is unlikely to be the case indefinitely, but presumably a firm will not often experiment in areas approaching the limit. The reasons why variable costs commonly appear to have what is usually called an 'L-shaped' average cost curve, but in fact have the 'U-shaped' curve of classical economic analysis, can be illustrated by a simple example. Suppose the empirical cost behaviour of a certain plant was:

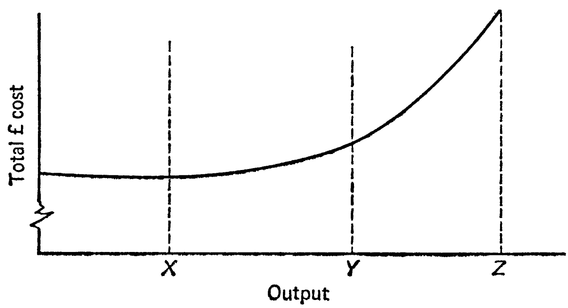


A regression line is constructed showing O, A as the unresponsive element of cost and the gradient of A, B as the responsive element. If this regression line is restated in terms of cost per unit of output, the cost of making very small quantities will approach infinity, while the unit cost will continuously approach nearer to the responsive element (but will never equal it) only as output is increased:

An L-shaped cost curve

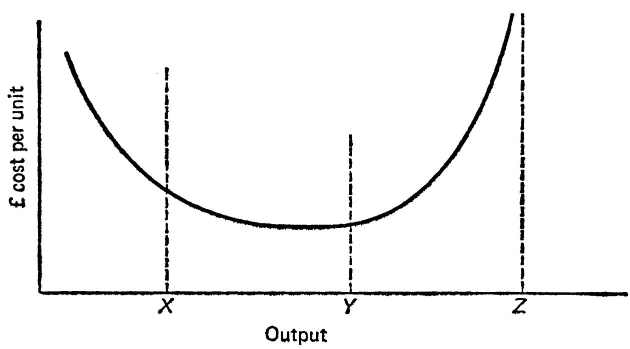


Of course the section of the lines lying between x and y on both diagrams are the only parts which are proven empirically. The 'real' cost function is quite likely to be non-linear, which is to say that the straight line A, B is not an adequate description of its behaviour. For example x might be the minimum output which could fully occupy the plant, so that the manufacture of smaller quantities could not reduce the total cost of the operations by very much. Similarly output cannot be extended above y indefinitely *at least within a limited time horizon*; most probably the upper ranges of any further feasible production must involve excessive maintenance and overtime working. The true total cost curve becomes:



and the cost per unit for this would be:

A U-shaped cost curve



If it is true that an enterprise only has real experience of production in the middle ranges of its cost-behaviour pattern, it is clear that some linear calculation may model its costs successfully within that range, while being highly incorrect beyond it. Above the range, cost per unit must rise eventually, but the area below this range is probably more dangerous, since it contains entirely feasible but unexplored levels of production. It can be seen that in these ways a firm may have feasible but unproven strategies in areas where cost behaviour needs to be carefully estimated from first principles, rather than assumed to be a linear continuation of known behaviour elsewhere. Empirical experience of production will not define necessarily the limits of the validity of a particular linear cost function. However the extrapolation of existing functions into unexplored ranges of activity requires to be justified as fully as the adoption of quite new patterns of cost behaviour for these ranges.

14. It will be observed that the terms '(linearly) responsive' and 'unresponsive' have been used in preference to more usual equivalents of 'variable' and 'fixed' in this section. Accountants usually take the overall view of costs, and hence see the origin of the regression line as the 'fixed element'. Unfortunately, some engineers, with equal logic, look at the problem from the unit cost itself, where it is the responsive element which remains constant, and the unresponsive element which is reduced as output is increased. As a result the meaning of fixed and variable is reversed for them, and dialogue between accountants and engineers is at times hindered by the use of the traditional terminology. However the earlier reference (in Note 7 of Chapter 1) to the paper by Ijiri and Itami (65) reminds us that cost behaviour is not really very easy to describe in the simple terms commonly used.

15. This statement is possibly true; unfortunately it is highly unlikely that time studies prepared in the ordinary course of business would be of much help here. The reason is that productivity bonuses and so on are negotiated nationally, but labour has to be recruited locally! Thus the wages paid cannot take account only of efficiency, but have to consider what a similar worker would obtain elsewhere in the locality. In short this is another area of business in which 'the books are cooked' in order to make the system work at all.

Of course this sort of bias is anathema to the academic specialist in job evaluation. One writer, T. T. Paterson, in his book, *Job Evaluation* (Business Books, 1972), emphasises the importance of leaving conditions of work and market factors out of the job content; his view is that these are the factors which are open to negotiation at shop-floor level, while the job content and its place on the enterprise's scale of remuneration 'should be as objective as possible' (p. 117). This approach is very interesting as an illustration of the material discussed in this chapter. This important element in manufacturing and service costs is seen as comprising definable negotiable and non-negotiable elements; presumably 'hard times' destroy market scarcities and make the most dubious conditions permitted by the law seem more tolerable, so as to force wages back on to the bare job-content range. On the other hand, one can see from the earlier discussion in this chapter that both management and workers may much prefer to shroud these matters in the decent obscurity of a global job-evaluation scheme, in practice.

5 The Building of Complex Statements

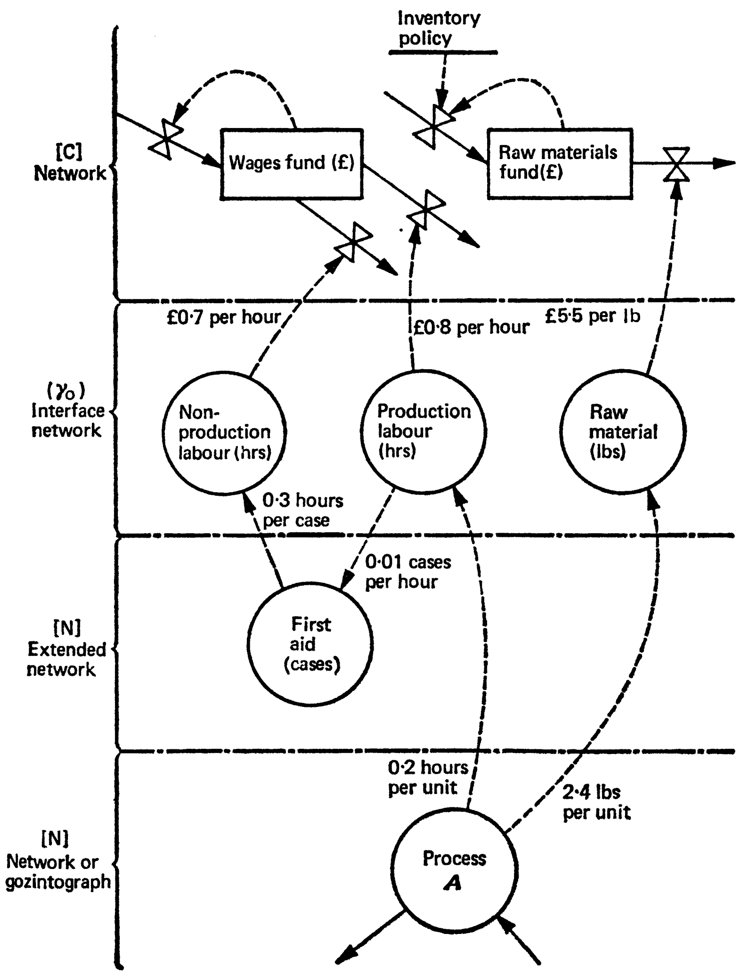
5.1 THE EXTENSION OF THE MATRIX [$\mathcal{N}$]

For the purposes of modelling the real system, the matrix [$\mathcal{N}$] was confined to a statement of the processes making up the real system and the physical flows which link them together. In Chapter 4 an attempt was made to show how these data formed part of a larger body of information which comprises the data bank of the information system of the organisation. That chapter also referred to the need for building a further model of this information system, principally as an aid to investigating the reliability of the complex statements which will be constructed out of the material in the data bank. This chapter will show the hierarchical nature of the processes by which the complex statements come into being; this suggests that the technological modelling technique itself might provide a good way of constructing a model of the information system also.[1]

To build this model, it is now necessary to include in the data bank some information about another flow which can be discerned within the real system, the flow of financial resources, or 'funds'. In general these data will be found under the 'cash' sub-section of the taxonomy, but it is obvious that these funds must have some sort of interface with the flow of physical production; this is the extension of [$\mathcal{N}$] or the 'nominal processes' which were referred to in the previous chapter. The normal conversion costs described in the earlier chapters were always the responsive (or variable) costs of production, so that it was the activity of the process which would determine the total normal cost of the process. It is convenient to add some further nodes and arcs to the original [$\mathcal{N}$], to represent the consumption by the process of its raw materials and production labour;

these nodes would be considered to have the generation number γ_0, so that basic processes would still bear γ_1. These nominal 'processes' can be thought of as being connected by linkages to the flow of funds; the linkages here are also arcs in a network, but carrying values such as '£ normal variable cost per unit of the activity of the following process', instead of the physical input-output coefficients of the gozintograph itself.

FIGURE 5.1 The interface of [C] and [$\mathcal{N}$][2]

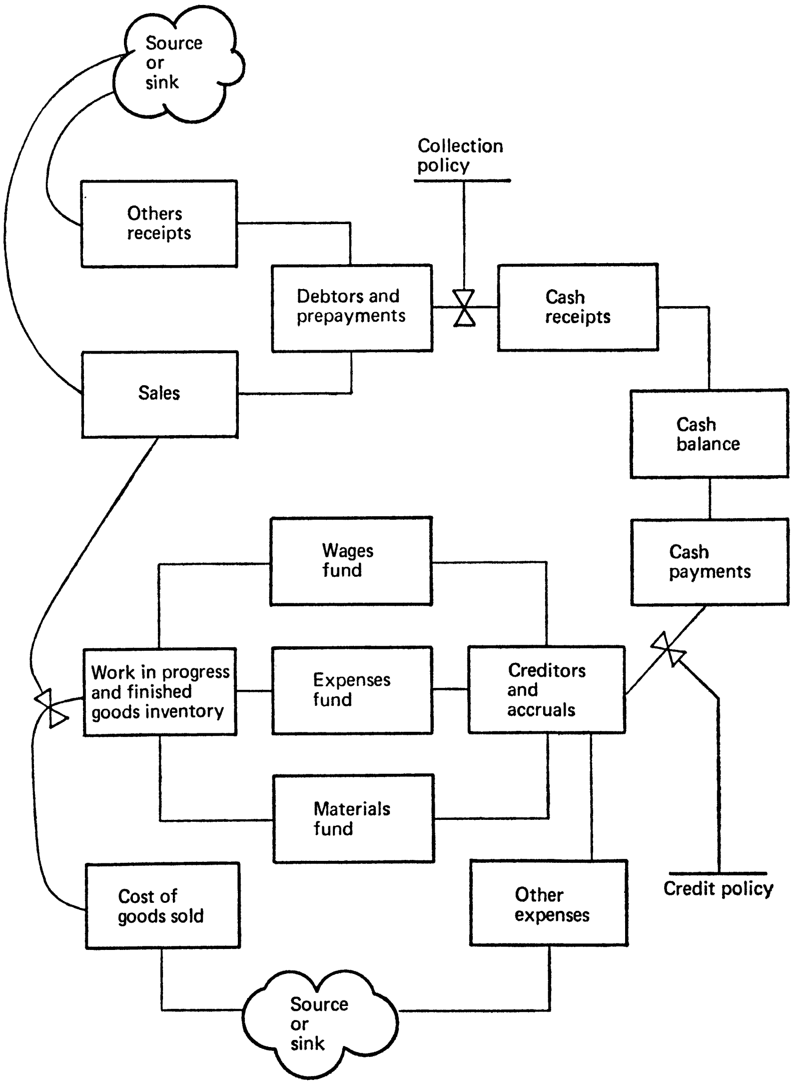


The nodes in the original $[\mathcal{N}]$-matrix represented processes with single homogeneous outputs; it has been seen that these processes are linked in some fashion (to be described in detail) with the underlying units of production plant in which they occur, and through them with the cost centres which compose that plant. So far it has been assumed that all the processes, and hence all the plant and component cost centres, formed a connected graph which was linked by physical flows of partly finished (i.e. intermediate) products. Consideration of an actual system will reveal that the network is almost always a disconnected graph; there will be a number of cost centres which attract costs in the ordinary way but are not connected by physical flows of intermediates to the main productive stream. These cost centres may also form part of larger groupings or departments in the same way that production centres form part of 'plants'. The disconnected cost centres are the service centres, and their costs form the works, administrative and selling-and-distribution overhead expense of the enterprise. These costs are semi-responsive costs, whose behaviour can be linked indirectly with the level of activity in the production centres or even the levels in other service centres. Accordingly these disconnected processes or departments are seen to be connected with some of the productive processes by another type of linkage, whose value will signify 'units of activity in the preceding node, per unit of output in the following node'. These linkages will not affect the γ-rating of the respective nodes. The overhead processes will interface with the flow of funds in the same way as the productive processes. These linkages are illustrated in Figure 5.1.

5.2 THE FINANCIAL ACCOUNTING SYSTEM

Section 5.1 has described the extensions needed in $[\mathcal{N}]$ to handle the interface with the flow of funds. In Chapter 1, the contents of $[\mathcal{N}]$ were equated with the 'Costing' part of the chart of accounts and the manual of procedures of the traditional accounting system, and the γ_0 nodes and the linkages just illustrated may also be considered as part of this chart and manual. The remainder of the chart and manual comprises the flow of funds system, which also can be represented as a series of arcs and nodes, and hence as matrix. This matrix could be named [*C*] and is the material described generally as 'Financial Accounting Procedures' in box (c), (1) of the taxonomy illustrated in Figure 4.1. Figure 5.2 is a simplified representation of this graph.

FIGURE 5.2 The financial accounting system[3] (for [*C*] network)



Unlike [*N*], this graph cannot be evaluated by attaching normal usages to the arcs. The flow out of any node in [*C*] is not dependent upon the activity of its immediate descendants, but rather upon the flow received from its immediate ancestor, plus or minus any change which has taken place, or is required to take place in the levels retained at most of the nodes. It can be seen that the matrix [*C*] differs from the matrix [*N*] in a number of other respects:

(a) The [*N*]-matrix shows the relative normal unit-for-unit activity of each pair of nodes in the physical flow and takes no specific note of changes in inventory. The [*C*]-matrix deals with total flows into and out of its nodes and so will always show cash balances and personal account balances at those points in the system.

(b) Moreover the [*N*]-matrix consists solely of normative data; there is no question of constructing the 'actual' version of this matrix to record the actual physical flows about the system. As was illustrated in Chapter 3, the variance in performance is calculated in respect of each process (and hence for each plant and cost centre), but these variances are used as a feedback of information for corrective action or subsequent amendment of future norms. Especially where economic accounting is employed, it is very rare for the actual cost per unit over all processes to be calculated for the final products. However [*C*] will run in parallel both normal data (for cash budgeting) and the actual data for the cash flow. Similarly the interface mechanism linking the fund accounts and γ_0 nodes will also carry both types of data, since it is necessary to calculate both the normal and actual costs of the cost centres to arrive at the variances.

Figure 5.2 contains arcs which show the flow of funds through the accounting system. It might be preferable to think of the relationships between the nodes of the [*C*]-matrix as undirected 'edges' rather than as directed arcs, so no arrowheads are attached to the flows shown in this figure. When recording the actual flows the debtors, receipts, cash balance, payments and creditors are all fixed historical data, so that the resulting (accrual-based) income and expenditure is in effect 'wound back' into the interface to produce a sales-analysis and the actual performance of the cost centres. In the budgeting operation, the required outputs and resources are wound forward into [C], and calculations of the budgeted debtors and creditors result in the budget for the cash balance.

5.3 THE LOWER-ORDER COMPLEX STATEMENTS

It can be seen from Figure 4.1 that the original input into the information system is 'historical data', which comprise both observations and authoritative statements. Omitting any problems which might arise over the analysis of the historical data itself at the moment of its absorption into the system, it might be possible to list the procedures necessary to use the new material in updating the old as follows:

(i) the modifications (if any) by the historical data of the normal data for the relative cost centres;
(ii) the redistribution of this normal data to plant, and thence into normal product or process conversion costs, normal capacities, etc.;
(iii) the calculation of normal responsive and unresponsive elements;
(iv) the calculation of normal overhead rates;
(v) the calculation of historical cash flow data

These operations will be needed whatever form of additional complex statement may be demanded from the information system, and so it seems worthwhile to distinguish these automatic ('lower-order') statements from the more advanced ('higher-order') forms which are not an essential part of the updating process, and so have to be called up specifically from the Library of Procedures if they are demanded. As has been explained in Chapter 4, the higher-order statements will always be built up from these lower-order statements, and never directly from the underlying basic statements.[4] The general form of the procedures required for lower-order statements would be as follows:

5.3.1 *The modification by the historical data of the normal data*

This operation is simply the editing (or smoothing) procedure described in the previous chapter, together with the monitoring (or bargaining) procedure which was also discussed therein; the result in either case is a new normal piece of data, representing an average of the most recent result and the previous normal cost, and containing, where appropriate, the normal negotiated element of organisational slack. Figures 5.1 and 5.2 have illustrated, in graph form, some finance-oriented extensions to the flow data which were the basis for the physical model described in the earlier chapters; these data are also to be found in the taxonomy of the information system of which they form part. However in the opening paragraph of this chapter it was suggested that it might be possible to make an additional model of the information system itself, using the same technique which the information system usually employs in the underlying real system. This can be illustrated by the construction of a further graph, which shows how the hierarchy of complex statements is built up from the basic statements. The convention to be used in these graphs is that the circular nodes represent basic statements, while complex statements of any order appear as hexagonal nodes.

At the outset a problem arises which is present in all discussions of 'information'; does one model what the agents themselves believe they are doing or what the model builder believes they are doing? (It has been suggested elsewhere that in general it may be an impracticable alternative to consider the agents' real activities as opposed to their own or other people's understanding of their activities.) Thus the agents probably do believe that they are smoothing the current observed total expenditure into the current total normal cost, and the result is then verified in some way by the budget controller, as Schiff and Lewin suggest. The analysis in the previous chapter suggested that more probably they *negotiate* a cost at some point above the normal rack cost of the operation.[5] With this approach the sole use of the total observed cost and the total normal cost (in this operation at least) is to provide some *status quo ante* for the bargaining operation. It will be assumed here that the latter analysis is correct and should be used in the model, since even if the former analysis is believed by the agents, in fact the 'verification' will consist of the negotiation procedure described by the latter analysis. The smoothing of the total costs may well occur, but it merely represents an updating of the *status quo* as a starting point for the negotiations.

This operation can be represented as another graph; however it will differ in some respects from the earlier figures in that it will not be possible to assign specific coefficients to many of the arcs. This is because the coefficients do not in general represent any underlying state of nature, but are themselves the results of a series of calculations dependent upon other factors. This means that it is not useful to construct a strictly static model of the information system. The model of the physical system which is presented by the information system at any one time is strictly static; on the other hand, Forrester's model is of course dynamic (op. cit.). However it will be preferable to consider our model of the information system as 'semi-static' and to include details of those calculations which depend on new inputs of data in the current period, while looking upon data produced within the system at earlier periods of time as if they were basic statements from outside.[6] Thus it will be useful to show the current values of the arcs in these graphs as 'rates' controlling the flow along them. The rates themselves can now be the subject of equations.

The procedures which are now to be described are necessarily complex and ill-defined – and hence debatable. This is because we are concerned with the way in which basic data are taken into the system and assimilated into lower-order complex statements, which are the modules for constructing the higher-order statements. However it would be a pity if this unavoidable difficulty were to obscure the fact that Lieberman charts and matrices are simple devices which can be constructed to model *any* clerical operation involving arithmetic.

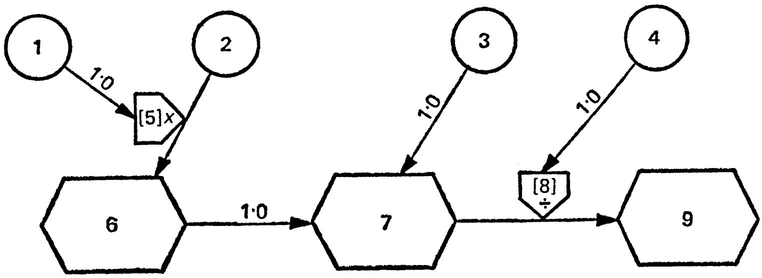
This can be illustrated by considering how some typical lower-order complex statements are assembled into an higher-order statement:

An overhead absorption rate is calculated as follows:

(3) Indirect material budget	£40
(2) Indirect labour hours 100 hours	
(1) Indirect labour rate £1·20 per hour	
(6) Indirect labour budget	£120
(7) Overhead budget	£160
(4) Estimate of direct labour hours 2000 hours	
(9) Overhead absorption rate per direct labour hour £0·08	

If one assumes that items 1, 2, 3 and 4 are basic statements (which normally they are not!) the resulting Lieberman chart becomes Figure 5.3.

FIGURE 5.3



This chart is seen to be semi-static in so far as some of the input-output coefficients are not directly evaluated, but require calculation; essentially these calculations will be multiplication and division and it is convenient to provide them with numbered nodes (here [5] and [8]). The resulting $[I+L]$-matrix cannot be inverted or evaluated in full until these calculations have been performed, but since the items are set in γ-order (with all 'basic statements' as γ_1) the necessary inputs for the calculations are always to hand:

		Basic statements				Complex statements				
		1	2	3	4	[5]	6	7	[8]	9
Basic statements	1	1				1				
	2		1				[5] *x*			
	3			1				1		
	4				1				1	
Complex statements	[5]					1				
	6						1	1		
	7							1		[8] ÷
	[8]								1	
	9									1

The technique is to (a) invert the matrix as far as the next calculation node and then (b) evaluate the inverse; this permits further inversion and further evaluation; thus:

$[I-L]^{-1}$

	1	2	3	4	[5]
1	1				1
2		1			
3			1		
4				1	
[5]					1

which is evaluated as:

	1	2	3	4	[5]
1	£1·2				£1·2
2		100 hrs			
3			£40		
4				2000 hrs	
[5]					0
	£1·2	100 hrs	£40	2000 hrs	£1·2

The next $[I-L]^{-1}$ and its evaluated version are:

	6	7	[8]
1			
2	1·2	1·2	
3		1	
4			1
[5]			
6	1	1	
7		1	
[8]			1

	6	7	[8]
1			
2	£120	£120	
3		£40	
4			2000 hrs
[5]			
6	0	0	
7		0	
[8]			0
	£120	£160	2000

The final vector of $[I-L]^{-1}$ can now be calculated and evaluated.

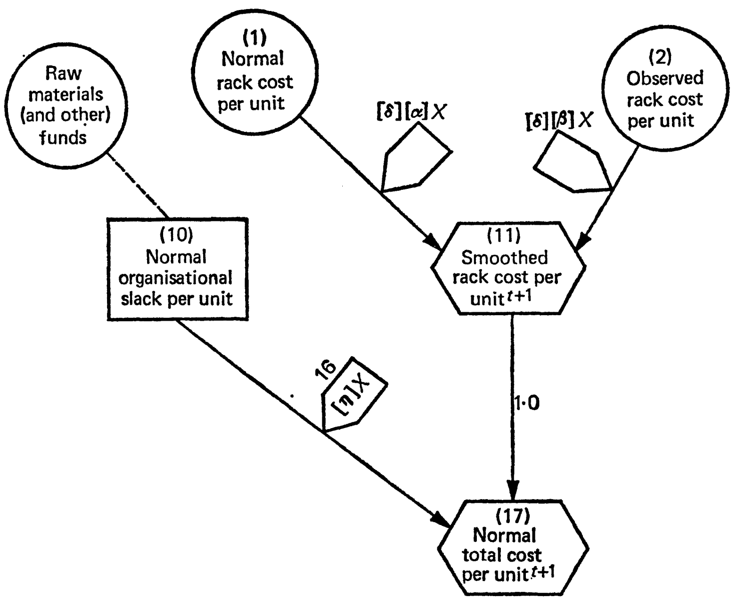
	9			9
1			1	
2	0·0006		2	£0·06
3	0·0005		3	£0·02
4			4	
[5]			[5]	
6	0·0005		6	0
7	0·0005		7	0
[8]			[8]	
9	1		9	0
				£0·08

It will be seen that only the 'basic statements' have inputs of value and the exogenous inputs of the complex statements are thus evaluated as zero. As has been said, the items (1)–(4) are really lower-order complex statements; perhaps the point is that a Lieberman chart can start at any level where one is prepared to accept some pieces of data as 'authoritative statements' whose values, probabilities and so on are not to be questioned!

The first part of the Lieberman chart for a typical lower-order complex statement might be Figure 5.4.

Here α and β are the constants referred to in Chapter 4 as the weights for the smoothing operation; at any one moment in time they will have some fixed values, but over time they might be expected to respond to some feedback of the experience of their success or failure in forecasting.[7] The coefficient δ, on the other hand, is the negotiated uplift added to the rack cost at each budget round. The graph will be extended later to show how this computed coefficient is arrived at. In general the normal organisational slack per unit for a cost centre might be expected to be that proportion of the difference between its estimated maximum cost per unit and its edited rack cost per unit, that its total edited slack in the previous period bore to the total edited slack of all the cost centres in the organisation; that is to say that the bargaining over the slack might be expected to confirm the *status quo*, as was discussed in the previous chapter.

FIGURE 5.4 Calculation of the normal cost per unit



However one would also expect some disturbance to occur here, on account of the non-economic, psychological, or gaming-strategy influences which might move the division of the estimated total slack available in the system away from the *status quo ante*; this will be designated ψ. The calculation of the normal organisational slack can now be expressed as

$$\eta = \frac{\chi\varphi}{E_i^{\,t+1}}$$

where χ = Estimated Total Revenue $^{t+i}$ $- \sum_{i}^{n}$ Total Smoothed Rack Cost $_i^{t+1}$
(χ is thus the total slack available in the next period $t+1$, Total

Smoothed Rack Cost $_i^{t+1}$ is the same as Smoothed Rack Cost per unit $_i^{t+1}$ multiplied by E_i^{t+1}, and E_i^{t+1} is the estimated activity of the ith cost centre in the next period.)

$$\varphi = \frac{\text{Actual Activity}_i^t\ (\text{Organisation Slack per Unit}_i^t + \psi_i^{t+1})}{\sum_i^n (\text{Actual Activity}_i^t)\ (\text{Organisation Slack per Unit}_i^t + \psi_i^{t+1})}$$

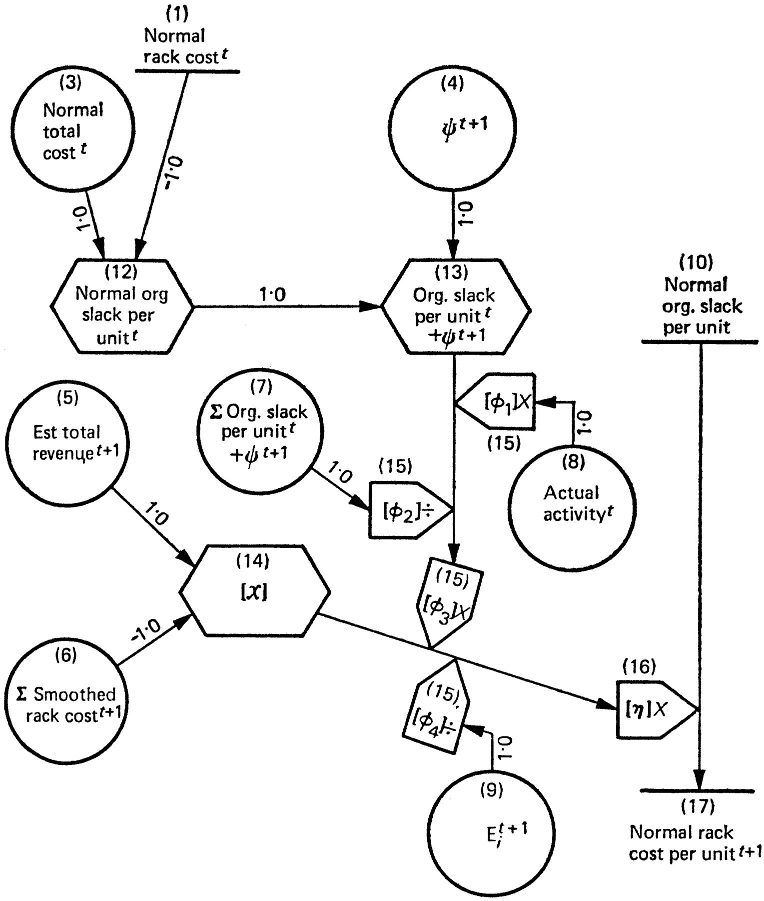
It is not proposed to examine the nature of ψ here;[8] one might remark that it is another interface, with yet another 'flow' (of sorts) to be found within the system. This is the human relations aspect, the psychological aspect or even the sociological aspect of the organism, which supplies the interface with the work of Fouraker and Siegel (41), Hofstede (59), and others who have concerned themselves with the bargaining 'Gaming' which occurs in business and industrial life. A further section of the graph of the information system model might be Figure 5.5 (note the treatment of the multiple operation in [φ]).

It will be seen that in effect this makes the Lieberman chart a series of disconnected graphs at least from an input-output sense.

In both Figure 5.4 and Figure 5.5 items such as 'Normal Cost *t*' have been shown as basic statements, although they must have arisen within the system, but in some previous period. This treatment is justified by the semi-static assumption of this model. To make Figure 5.5 a little easier to understand, Nodes (7) and (8) have been included to show the inclusion of summations for all cost centres in the calculations; this will create few problems, since the endogenous components of these items will be the same as those of the ith centre itself.

This analysis has implied that the organisational slack is taken up into the cost structure as an addition to the responsive conversion costs of the various cost centres. If so it must also affect the interface machinery illustrated in Figure 5.1 (a), since the slack will take the form of additional usages of resources through the γ_0 nodes.[9] Nevertheless it seems reasonable to measure slack initially in money terms; provided the constrained resources are being used efficiently, the management side of the bargaining team should logically be indifferent as to where the slack is to be located so long as the overall cost does not exceed the limit which would permit a satisfactory return to shareholders and allow for adequate expansion.

FIGURE 5.5 The calculation of [η]



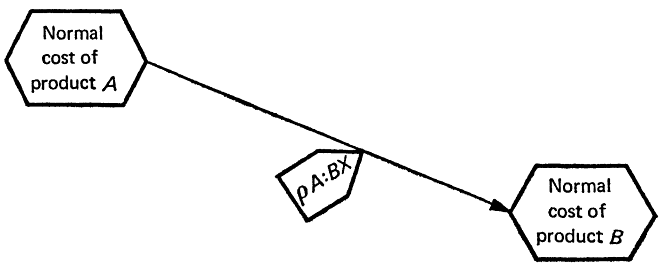
5.3.2 *The redistribution of the normal data*

The procedure described in sub-section 5.3.1 has thus produced for each cost centre, a normal exogenous cost for use in the forthcoming budget period. Further procedures will be needed to relate these normal costs (a) to their respective plants and thence to the processes and (b) to some normal usages for γ_0 inputs of raw materials, etc. Rather similar procedures will be needed to relate the capacity data, which arise originally in the various cost centres, to the processes and products. The summation of cost-centre data into plant data does not involve the creation of further more complex statements, since the latter are simply a series of sets of which the former are the elements. The position is different in the case of the multi-product plant. Generally cost centres will have different rates of throughput and hence differing costs for the various products which they process. If these separate costs could be negotiated for each product, it would only be necessary to find some common denominator of activity which would link these separate throughputs within some overall measure of capacity for the cost centre. However where batch production occurs in multi-product plant, there are often only very indistinct indications of what these separate costs may be. Accordingly a common practice is to select one product as the basic product of the plant and then express all the alternative outputs as '(equivalent) ratio units' of the basic product. This means that all the data for the plant are to be expressed in terms of the basic product, and the capacities and process costs of the other products are arrived at by multiplying this data through by a scalar amount representing the 'ratio units'-to-one for the particular product.

The ratio units could be renegotiated at the budget in the same fashion as the costs themselves; possibly they might present another opportunity for organisational slack. However since it is the essential uncertainty over the true relationship of costs to products in these cases which gives rise to the practice, it seems improbable that meaningful negotiation on the lines discussed in Chapter 4 could occur. It is usually suggested that the existing *status quo ante* would prove a very powerful influence where the parties to a dispute do not have any reliable data to guide their decisions (and they are evenly matched psychologically) and personal observation would suggest that these ratio-unit figures tend to remain unamended for very long periods of time; perhaps one of the reasons for the existence of organisational slack is to enable a supervisor and his subordinates to live with the uncertainty introduced into their tasks by multiple production. Of course the ratio-unit system can only apply where the basic product is predominant in the operations of the plant; if one of the other products becomes produced more frequently, so that the inaccuracy in the ratio becomes important, the supervisors will demand a specific costing for the product, and moreover the necessary data for this will have become available.

Where ratio units are employed the redistribution of data will involve the creation of a further more complex statement from the conversion cost of the basic product. The ratio unit scalar is simply a constant, ρ_{ij}, which is also susceptible to change by a feedback from the success or failure of the system in forecasting its costs in the plant as a whole, over the long run. Thus:

FIGURE 5.6 Calculation of normal cost in the multi-product plant



The principal significance of this type of operation will become apparent in Chapter 6, when the problems of the reliability and probability of the data in the information system will be discussed. The intervention of ρ_{ij} will usually make the cost of product B less 'reliable' than that of product A, the basic product, as well as affect any measures of probability which can be attached to costs of this type.

5.3.3 *Responsive and unresponsive elements of cost*

The need to prepare forecasts of future behaviour of the real system implies the division of normal costs (and usages) into two elements which are responsive or unresponsive to changes in output. A footnote to Chapter 4 showed how this response was seen, for the most part, as a *linear* response which so produces the stable 'variable cost per unit' which is a feature of the direct or marginal costing system. The basic observations picked up as 'historical data' will themselves form part of the regression operation by which the division is recalculated. It is necessary to consider the relation of this regression to the smoothing operation which has been described already. The observation of the actual cost will be in the form of a total ('fixed' + 'variable' + 'slack') cost for some level of output. If no part of this total is judged to be a controllable variance, this figure will be taken up into the regression to produce a new actual unresponsive element for the cost and a new *unit* responsive element, and it could be argued that a regression over *n* observations is itself a form of exponential smoothing.

However if the organisational slack is being measured also, the situation is more complex. The 'observed rack cost' is likely to be expressed originally in the form of a rate per unit and will either be linearly responsive for all levels of output, or have its non-linear behaviour specified in some way, and so any unresponsive element will also be specified. Thus the observed rack cost can be smoothed directly into the normal rack cost in the normal way, without regard for the total actual cost per unit. The total observed organisation slack can be ascertained by calculating the total observed rack cost of the actual level of output and deducting this from the total actual cost. As has been explained, it makes little sense to attempt to split slack cost into responsive and unresponsive elements, so the observed organisational slack per unit is arrived at by simply dividing the total observed organisational slack by the actual output.[10]

5.3.4 *The calculation of overhead rates*

The foregoing remarks will apply to all costs, whether they are directly connected with the physical flow of production through the enterprise, or related only through the overhead linkages which have been described in Section 5.1; the parameter on the linkage in the latter instance will result from a regression operation also.

Organisational slack is not responsive to technical necessities but is seen as an inflation of the responsive element of the rack cost of the centre which is *permitted* to occur. Accordingly it seems reasonable to calculate the overhead expense regressions in terms of the rack costs of the service centre, so that the resulting linkage would reflect the true technological relationship, which would not be disturbed by any voluntary inflation of the service centre's costs. Here too the regression operation would provide an element of smoothing to the normal linkage coefficient.

5.3.5 *Historical cash flow data*

As well as maintaining and updating the normal data in the system itself, the information system will also update the historical records of the financial transactions of the organisation. So far the only aspect of historical accounting which has been referred to is the 'winding back' of the observed expenditure into the interface mechanism so as to allocate it to the appropriate cost centres, prior to its employment in the recalculation of normal costs. This 'observed expenditure' is the accruals-based expenditure of conventional accounting; by means of the housekeeping data these items can be reanalysed and wound forward into the creditors and accruals 'inventory'. This inventory is in turn reduced by the further historical data which will have been maintained of the payments of cash; similar records will be drawn from the sales to produce the historical net cash balance as the final level reached by the two flows. It may be that some day-to-day control operations should be included here (see Note 6 to Chapter 7).

5.4 SOME ASPECTS OF DATA HOUSEKEEPING

In describing the model from the point of view of its employment in the solution of a variety of accounting and planning problems, one should not overlook the fact that the construction, updating and use of 'data banks' of all kinds is at the heart of the discipline of librarianship, in its widest sense. Perhaps one could include the algorithms which range the technological data in generation order, and those for the inversion of the data, under this heading; also this might cover the procedures for locating and collating the code numbers and other identifying data needed in the information system.

An important aspect of large-scale data processing is the need to keep the data bank in as uncorrupted a state as possible. When the structure of the underlying real system changes through the discontinuance of a product or intermediate, it is essential to ensure that all the products which use the discontinued item are either themselves discontinued or can accept a substitute input for that item; the rows of the [*T*]-matrix will supply a list of the affected user-processes. However especially where output is in the form of batch production, where some products may be produced only very infrequently, it is possible to overlook some of the secondary and tertiary effects of such changes. The discontinuance of a product of generation γ_i may not only lead to the redundancy of one or more items in γ_{i+1} but also lead to the discontinuance of further products in γ_{i+2} and so on. Similarly the discontinued product may be the sole *user* of products in γ_{i-1} and thus initiate a chain of redundancies amongst its ancestors also. The system needs an algorithm which will take lists of the ingredients and usages of a redundant product, and then sequentially hunt out any consequent chains of redundancy. Obviously a search of the rows and columns of [*T*] (rather than [*N*]) will provide this.

Another housekeeping function within the data processing system will be to take some account of the timeliness of the data in the bank. The distinction between forecasting and current information can never be absolute, since in practice new pieces of basic data will arrive at random intervals of time; this means that at any one time, t, the contents of the bank will not necessarily be homogeneous in the sense that it consists of normal data which have been smoothed up to time t, or indeed up to any one $(t - i)$! If out-of-date information is used in place of current material, an extrapolation is being made on the basis that:

$$\bar{x}_t = \bar{x}_{t-i}$$

Since the data bank has been assumed to contain data necessary to forecast future states of the variables, it would seem preferable to make a specific forecast of $\bar{x}_t$, based on the position at $t - i$, and perhaps take account of the decreasing reliability of a piece of data over the passage of time. Unless it is assumed that out-of-date information *is* a rough forecast of some sort, complex statements built up from unequally-updated basic statements are strictly meaningless. This part of the library routines of the data bank is analogous to the 'clock' in a real-time data processing system, which at any time will cause the new basic data which has accumulated since $t - 1$ to be taken up into the normal cost data, and then substitute forecasts for any items which have not been currently updated.

A large part of the library procedures will be concerned with the physical storage of the various data. Probably many of the users of the information system will have only limited areas in which they frequently require data. For example where an enterprise is composed of a number of departments, much of the information will be needed only by the local managers; in turn these managers will rarely have much cause to consult the data relating to any departments excepting their own. It is possible to reduce the average time taken to find a piece of data (its 'access time') by taking account of these considerations, and segregating locally-used material into special areas of the store. Similar problems arise over the initial division of the store and the quantities of spare accommodation to be left for expansion in any one location, and over the most efficient system for the allocation of empty locations which may arise when redundant structural data are removed.

The location and retrieval of data will be closely connected with problems of security. Much of the data in the data bank will be classified to varying degrees of confidentiality, to be made available only to certain persons or classes of persons. Obviously this aspect of the information system has received especial emphasis in military intelligence applications (25).[11]

5.5 THE QUANTIFICATION OF THE INFORMATION MODEL

The figures illustrating this chapter are graphs which are similar to 'gozintographs', showing the flow of information through the information system, very much in the way that earlier graphs demonstrated the flow of products and intermediates through the physical system. The coefficients on the arcs in the information graphs have sometimes been given as variables such as α, β, η and φ, because these coefficients are liable to change as a result of current exogenous inputs into the system, or feedback from their success or failure as predictors of the behaviour of the system. If the system is assumed to be semi-static, in the sense defined previously, these coefficients will have numerical values which will indicate the relative weights given to the ancestor statement in the compilation of the receiving statement. As a result it becomes feasible to construct 'next-assembly' and 'total-assembly' matrices which show the input-output relationships of the information system, using techniques similar to those which have been developed in earlier chapters for the physical system. It will be convenient to describe these graphs and matrices as 'Lieberman charts' and 'Lieberman matrices', or $[L]$ matrices,[12] so as to reserve the terms 'gozintograph', '$[N]$' and '$[T]$' for their physical counterparts only.

Returning to the examples in Figures 5.4 and 5.5, the construction of its static Lieberman chart and matrix will require the evaluation of the variables. This might be done purely subjectively, on the basis of some assumptions about the relative 'importance' to be attached to some input. However since the operations being described in the chart represent definite arithmetic calculations of addition and multiplication, it is apparent that the assignment of subjective weightings itself assumes that α, β, etc. must have certain values. Moreover it is necessary to decide whether the intention is to arrive at the relative influence of the different factors upon the output or to record the relative composition of that output. For example, if $\bar{X}$ is the weighted average of X_1, X_2 and X_3 with weights of 3:2:1, then $\bar{X}$ comprises $\frac{1}{2}X_1$; $\frac{1}{3}X_2$; $\frac{1}{6}X_3$ and these are the relative influences of the factors.

On the other hand, if $X_1 = 1100$; $X_2 = 1200$; $X_3 = 300$, the calculation becomes:

$$\begin{aligned} X_1\ \tfrac{1}{2} \times 1100 &= 550 \\ X_2\ \tfrac{1}{3} \times 1200 &= 400 \\ X_3\ \tfrac{1}{6} \times 300 &= 50 \\ \bar{X} \qquad\quad &= \overline{1000} \end{aligned}$$

It is now possible to say that the relative contributions to the value of $\bar{X}$ from X_1, X_2 and X_3 are 55 per cent, 40 per cent and 5 per cent respectively, in so far that these measure the amount they contribute to the final result. Clearly the first interpretation is on the same lines as the technological model, which states that 1 lb. of product A requires inputs of 0·5 of a gallon of B, 0·3 of a lb. of C and 0·16 of a pint of D; supposing that the concern was to establish the responsive cost of product A, the relevance of this information to the result would be dependent also upon the variable cost of B, C and D. Thus Figure 5.6 shows that 'smoothed organisation slack' + ψ'+1' has one unit of ψ as part of its input; since ψ can be any negative or positive amount which reflects the effect of the interface with the 'psychological' system of the organisation, this statement tells the enquirer nothing he really wants to know about the nature of the complex statement he is considering. Fortunately in the technological model the price of a product can be ascertained by multiplying its usage of the ancestor processes by their conversion costs; for the Lieberman chart too the composition of the result can be arrived at by multiplying the coefficients showing the relative influence of the various basic statements by their value.

A matrix can be constructed in the usual way from the graph appearing as Figures 5.4 and 5.5; if the nodes are placed in their correct ancestor/descendant relationship, this will exhibit the normal upper-triangular form. As has been mentioned before, it is possible to construct a static model where the variable coefficients are seen as being fixed externally. Where these coefficients vary because of feedback from earlier periods of time, this is correct, but for most purposes it will be necessary to calculate the values for the variable coefficients which depend solely upon the values of various levels in the current period. Thus it would be appropriate for the present illustration to assume these values: $\delta = 0{\cdot}9$; $\alpha = 0{\cdot}2$; $\beta = 0{\cdot}8$; and hence that $[\delta]\,[\alpha] = 0{\cdot}18$ and $[\delta]\,[\beta] = 0{\cdot}72$.

However in order to calculate φ and η it is first necessary to construct the matrix as far as node (15), invert it, and evaluate it, as in the earlier example. This is because the graph is disconnected (twice) at that point and it is necessary to provide the righthand side of the equations. The uninverted Lieberman matrix $[I+L]$ for this is Matrix 5.1.

The Lieberman matrix can now be inverted and evaluated as far as $\eta[16]$ (Matrix 5.2).

The Lieberman matrix is divided into quadrants, as between its basic statements and its complex statements. The significant area is the upper righthand quadrant, which maps the basic statements into the complex statements. Continuing the techniques used on the technological model, the matrix can now be

Matrix 5.1

		Basic statements										Complex statements									
		(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	[15_1]	[15_2]	[15_3]	[15_4]	[16]	(17)
Basic statements	(1) Normal rack cost^t	1										0·72	−1·0								
	(2) Observed rack cost^t		1									0·18									
	(3) Normal total cost^t			1									1·0								
	(4) ψ^{t+1}				1									1·0							
	(5) Est. total revenue^{t+1}					1									1·0						
	(6) ΣSmoothed rack cost^{t+1}						1								−1·0						
	(7) ΣSmoothed org. slack$+\psi^{t+1}$							1									1·0				
	(8) Actual activity^t								1							1·0					
	(9) E_i^{t+1}									1									1·0		
	(10) Normal org. slack per unit										1										[16×]
Complex statements	(11) Smoothed rack cost^{t+1}											1									1·0
	(12) Normal org. slack^t												1	0·72							
	(13) Smoothed org. slack$+\psi^{t+1}$													1				[15_1× 15_2÷]			
	(14) χ														1					[15_3× 15_4÷]	
	[15] $\varphi_1, \varphi_2, \varphi_3, \varphi_4$															1	1	1	1		
	[16] η																			1	
	(17) Normal total cost per unit^{t+1}																				1

Matrix 5.2

		Basic statements										Complex statements								
		(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	$[15_1]$	$[15_2]$	$[15_3]$	$[15_4]$	[16]
Basic statements	(1)	1										0·72	−1·0	−0·72						
	(2)		1									0·18								
	(3)			1									1·0	0·72						
	(4)				1									1·0						
	(5)					1									1·0					
	(6)						1								−1·0					
	(7)							1									1·0			
	(8)								1							1·0				
	(9)									1									1·0	
	(10)										1									
Complex statements	(11)											1								
	(12)												1	0·72						
	(13)													1				$\left[\begin{matrix}15_1\times\\15_2\div\end{matrix}\right]$		
	(14)														1					$\left[\begin{matrix}15_3\times\\15_4\div\end{matrix}\right]$
	[15]															1	1	1	1	
	[16]																			1

Matrix 5.3

	Basic statements (1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	Complex statements (11)	(12)	(13)	(14)	[15_1]	[15_2]	[15_3]*	[15_4]	[16]†
Basic statements (1)	£4·00										2·88	−4·00	−2·88						
(2)		£5·00									0·90								
(3)			£7·00									7·00	5·04						
(4)				0·30									0·03						
(5)					£75,000									75,000					
(6)						£50,000								−50,000					
(7)							£20,000									20,000			
(8)								5,000							5,000				
(9)									6,000									6,000	
(10)										£1·00									
Complex statements (11)											0								
(12)												0	0						
(13)													0				[$15_1\times$ $15_2\div$]		
(14)														0					[$15_3\times$ $15_4\div$]
[15]															0	0	0	0	
[16]																			0
Value of the complex statements											£3·78	£3·00	2·46	£25,000	5,000	£20,000	*	6,000	†

* The coefficient φ_3 can now be calculated:

(i) $(13)\times[15_1\times]=2{\cdot}46\times 5{,}000=12{,}300$

(ii) $\div[15_2\div] \qquad =\dfrac{12{,}300}{20{,}000} \qquad = 0{\cdot}615$

† and η thus becomes:

(iii) $(14)\times[15_3\times]=25{,}000\times 0{\cdot}615=15{,}375$

(iv) $\div[15_4\div] \qquad =\dfrac{15{,}375}{6{,}000} \qquad = 2{\cdot}562$

evaluated, assuming the following values for the basic statements:

(1)	£4·00	(4)	+ 0·30	(8)	5000 units		
(2)	£5·00	(5)	£75,000	(9)	6000 units		
(3)	£7·00	(6)	£50,000	(10)	£1·00 (because it has to		
		(7)	£20,000		be evaluated, in £'s by η)		

The intrinsic value of a complex statement is always nil, as we have seen (Matrix 5.3).

Column (17) (normal total cost per unit $^{t+1}$) can now be inverted and evaluated:

	(17)	(17)
(1)	0·72	£2·88
(2)	0·18	£0·90
.		
.		
.		
(10)	2·562	£2·562
(11)	1	0
.		
.		
.		
.		
(17)	1	0
		£6·342

The relative contributions of the basic statements can be calculated from the top righthand quadrant of the evaluated $[I-L]^{-1}$-matrix as shown on page 151.

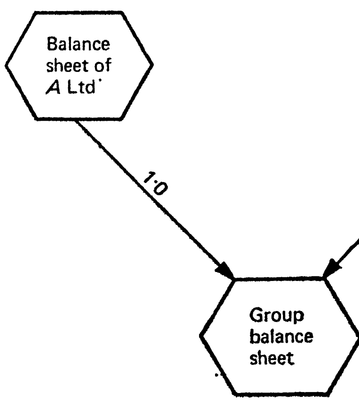
This method of calculation is debatable even in terms of the static model. Although basic statements (7), (8) and (9) for example do not directly form part of the value of any complex statement, they have played a part in the calculation of φ, as did (3), (4), (5) and (6). Basic statement (10) has contributed 40 per cent to the total of (17), but only through multiplication by η. The fact of the matter is that the Lieberman chart is typically a *disconnected graph.* As can be seen in Figure 5.4, basic statements (1), (2) and (10) alone have an input-output relationship with complex statement (17).[13]

	(11)	(12)	(13)	(14)	(17)
(1)	+76%	- 133%	- 113%		+ 45%
(2)	+24%				+ 15%
(3)		+233%	+205%		
(4)			+12%		
(5)				+ 300%	
(6)				- 200%	
(7)					
(8)					
(9)					
(10)					+ 40%

5.6 HIGHER-ORDER COMPLEX STATEMENTS

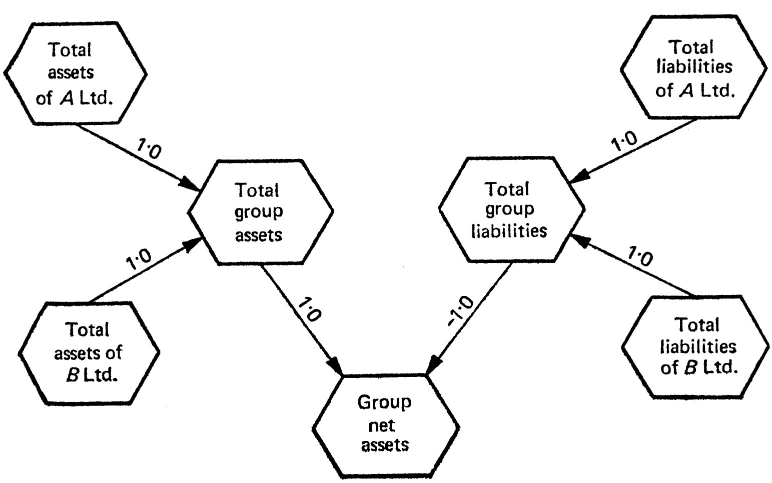
More complex statements will all be based upon the updated normal and actual data carried in the data bank. These can be assembled and analysed in the same fashion as the lower-order statements, as was shown in Figure 5.3. The operation is seen as a series of flows through a system, and so the mathematical operations of addition and subtraction, and also multiplication and division by constant scalars, can proceed sequentially as a normal assembly process. It is only when a statement has to be multiplied by a variable amount that it becomes necessary to invert and evaluate the matrix at an intermediate point, as illustrated in the preceding sub-section. Notice that the chart considers individual arithmetic calculations rather than the overall preparation of documents. In a highly simplified example, it is *not* possible to say:

FIGURE 5.7 (a)



Instead this would have to be arrived at, say, as Figure 5.7 (b).

FIGURE 5.7 (b)



5·7 NOTES

1. It is relevant to discuss whether the material in this book is to be seen as an exercise in theoretical model building, or as something approaching the block diagrams for a computable information system. In fact the material described in Chapters 2 and 3 has formed the basis of operational systems for modelling the technical flows in process manufacture; the remainder of the book is an attempt at a formal analysis of the whole informational system. Various parts of the interface between [N] and [C] have of course been successfully 'automated' in practice, as have parts of [C] itself, but the whole of the proposed scheme would comprise a fully integrated total system. It is unlikely that such a system could be automated successfully in its entirety at this time; as will be seen in Chapter 7, the nature of the feedbacks within such a system is not well understood, but it may be assumed that it is exceedingly complex.

2. The convention of this graph owes something to Forrester's book (op. cit.). The nodes in the interface network are what he would call 'auxiliary equations'; they control the rate of the flows into the fund accounts. The fund accounts themselves are designated by rectangles, which means they are levels which are capable of carrying inventories on balances. The outflows from the funds are governed by the level of the fund and the policy over inventory or balances (if any).

3. Figure 5.2 is described as 'the financial accounting system', as distinguished from the costing system; it just shows the flow of 'funds' and cash into and out of an internal sink/source. It does *not* therefore attempt to show how 'the financial accounts' or 'the cost statements' are prepared, because such activities are not part of the flows shown in Figure 4.1. Rather they are prepared from data extracted from the flows – which is why we shall be referring to them as 'complex statements'.

4. This distinction illustrates and reinforces the statement made in Note 1 to Chapter 4. The data maintained in the 'data bank' are kept as these normative, lower-order complex statements. This provides the official corporate *Weltanschauung*: at any one time we have an accepted opinion about what our costs, capacities and so on are, and we make our decisions accordingly – usually with the aid of higher-order statements of some kind.

5. The normal rack cost used here is arrived at by smoothing the currently observed rack cost into the previously held normal rack cost. It is being assumed that the rack cost will vary from period to period although in general it is probably more stable than the total cost figures. The principal advantage of using behavioural concepts in the analysis of cost behaviour is that it eliminates all psychological and 'human relations' influences on cost as organisational slack, and leaves the rack cost as the element which is influenced only by physical, or technological, considerations. Thus the normal cost is arrived at by both smoothing *and* negotiation. The rack cost is internally generated and so could be the object of a secret reserve – if its existence was admitted openly by the negotiants. (See Note 9 of Chapter 4.)

6. The reason is simply that all basic data are assumed to be statistically independent. If one considers the fact that much of our data has been

generated internally over time, it becomes highly interrelated and introduces the complexities of auto-correlation into our calculus of probabilities. It will be apparent that this exposition will become sufficiently complex without that! On the other hand, the auto-correlation really does exist. What this implies is the subject of the latter sections of Chapter 7; briefly we would be attempting to develop a stochastic version of a dynamic model.

7. The mechanics of this part of the system are envisaged for the moment to be as follows: $\alpha + \beta = 1$, while δ is some decimal fraction representing the change expected over the period t to $t + 1$. Both α and β are multiplied though by this factor δ, so that $[\delta]\ [\alpha] + [\delta]\ [\beta] = \delta$. Of course this assumption of linearity is rather simple-minded!

8. Something is said about ψ in Section 7.3 of this book and also in *Societal Accounting* (op. cit.), especially in Section 7.2.

9. If slack was permitted in the endogenous usages of intermediates by other processes, this would give rise to additional activity within the real system and so create further organisational slack. This is probably the reason that most enterprises seem to treat endogenous usages as rack costs and permit no organisational slack in the use of internally produced resources. There would be no logic in this distinction, except for the fact that the overall incidence of organisational slack would be very difficult to measure or control if it were allowed to occur in this area.

10. In Figure 5.4 'the observed rack cost' has been shown as a basic statement, notwithstanding the fact that it must have been arrived at in the way described here. In fact the act of 'observation' always implies some elements of recognition, analysis and measurement, which may apply to different 'aspects' of the phenomenon being observed. The problems arising from this will be discussed in Chapter 6. There is no question of smoothing the slack of course. Variances in slack are by definition controllable variances and vice versa. We shall see in Chapter 7 that such 'bargains' should be kept, except to the extent that they prove too generous or too harsh in operation!

11. It may not be desirable to tell personnel precisely what limits are set to their own access to information in the bank, since this may suggest areas for the exercise of untoward curiosity. In the system described in (25), an attempt to withdraw classified information by an unauthorised person is reported to his next superior officer who has access to it; the device is not essentially punitive, since one interpretation of the demand is simply that someone is tackling a problem which really requires the assistance of a superior!

12. So called after the pioneering paper by I. J. Lieberman (73).

13. Lieberman's own methodology would show that all fourteen earlier statements were inputs of (15). This is because he is not concerned with the nature of the arithmetic operations, but only with the presence or absence of a statement in the function; thus his matrix is a normal input-output matrix and is not disconnected. However we shall see in Chapter 6 that the nature of the operation will usually have a profound effect upon what we want to know about the data in an information system.

6 The Probability and Reliability of Complex Statements

6.1 THE ACCURACY OF THE BASIC DATA

In earlier chapters several references have been made to the paradox that while almost all practical work in accounting and even in quantitative decision making assumes that the data used as inputs are deterministic, the reaction of the users of the outputs of these exercises indicates a certain scepticism towards the results, which can be attributed to a belief that the facts of the real underlying system may differ more or less substantially from their reported condition. The recent and well publicised errors in the national export statistics are just a further example which illustrates the point.

There might seem to be a danger that a board of directors or even a government might be induced to take quite the wrong action, on the strength of very sophisticated analyses of summaries of basic data whose reliability is patently suspect in their original form, but accumulated experience of this phenomenon often makes such bodies surprisingly lethargic in the face of the 'facts' which are placed before them. Attempts are sometimes made to rationalise this attaching of more credibility to a summation than to any individual piece of data which comprises it, by an appeal to a Law of Very Large Numbers which suggests that in a summation the individual errors tend to cancel one another out, so that the more observations that go into a total, the nearer the observed total will approach the true total. Now this is coming very near to the heart of the matter; the Law is true enough when unbiased readings for homogeneous phenomena are being summed. Unfortunately, as has been demonstrated, the 'readings' contained in the information system are usually neither unbiased nor homogeneous.

It is futile to apply very fine tools of analysis to faulty data. In such circumstances profound changes in the results may arise from quite small variations in the data, which could easily be explained away as of no consequence in view of the range of tolerance demanded by their unreliability.

Oskar Morgenstern's book, *On the Accuracy of Economic Observations* (83), is devoted to establishing this point; the book is well known, but few of its lessons seem to be applied in practice! Professor Morgenstern deals with accounting only briefly, since his main concern is with the use of over-fine analysis in macro-economics; however he emphasises the need to include some statement of the distribution of the data presented in all accounting and statistical statements, as some guide to their reliability. It is apparent that the complicated construction of most accounting statements is likely to make it difficult to establish guidelines of this sort, but before considering some technique which might be of help it is necessary to say something about the current state of applied statistical decision theory, in order to distinguish those aspects of the 'theory of probability' which are discussed therein from the difficulties which will be examined in this chapter.

6.2 APPLIED STATISTICAL DECISION THEORY

The circumstances which are assumed as a basis for statistical decision theory are the following: (a) The decision maker is faced with a choice amongst various courses of action, (b) where the result of any one course of action depends upon the actual 'state of the world' and (c) where it is possible to obtain more information about that state of the world, by carrying out what are described as 'experiments' whose outcomes will indicate weights which the decision maker should attach to the various possible states of the world. This implies the existence of a 'joint probability measure' for each experiment, which is the probability that some particular state of the world will in fact apply, given that some particular experiment has been carried out with some particular result. This joint measure clearly links four interrelated probabilities:[1]

(a) The marginal probability that this state of the world will in fact be the real state of the world, which the decision maker would assign *prior* to knowing the outcome of the given experiment
(b) The conditional probability that this outcome will result given a particular experiment and actual state of the world
(c) The marginal probability that the outcome would follow the given experiment, whatever the real state of the world
(d) The conditional probability that some state of the world will be the real state of the world, which the decision maker would assign, *after* the experiment has been carried out and the result known (i.e. the *posterior* probability)

The remainder of the theory does not directly concern the argument of this book. However if it is accepted that this type of analysis sufficiently defines the decisions which have to be made in the course of business affairs, it is necessary to ask (a) what results of which experiments are to be found in the information system which has been described in the earlier chapters of this book and (b) how can the decision maker extract sufficient data about the marginal and conditional probabilities listed above, to enable him to arrive at the necessary joint probability measure. Question (a) was in part answered in the previous chapter; it would be possible to make a specific experiment and use its results, but in practice it is usual to extract 'surrogates' for these results from historical observations and external authorities, in the fashion described above. Clearly the writers on decision theory are normally discussing some fairly self-contained problem, like taking a sample from a batch of identical components, and accepting or rejecting the whole batch on the number of rejects discovered in the sample. In such a case the real state of the world is that the batch is in fact either of an acceptable quality or not, and the result of the experiment may be supposed to give some indication of this. Many decisions facing management are likely to be more complex than this simple sampling operation, where the states of the world which will affect the outcome of trading activity will be complicated entities such as the demand for the product, or the cost of its production.

Econometric modelling has demonstrated that it is possible to have an *a priori* 'model' which might suggest directly or indirectly what that demand or cost might be. It may be possible to see the subsequent 'experiment' and 'result' as the accounting operations and their output, on the basis of which the decision maker revises his 'model' of the state of the world. More precisely he will revise the measure of probability which he attaches to the various feasible states of the world, on the basis of the feedback of results from using the earlier model, which will be reflected in the α, β, δ and ψ he uses to arrive at his 'normal' data (see Chapter 7). Since it can only be the information system which can supply the *a priori* measures of probability (since this system can comprehend *ex cathedra* beliefs as 'authoritative statements'), the 'experiments' in this case must be the updating processes and the 'results' will be the variances between budgeted and normal performances and between normal and actual performances.

So far this book has assumed that only one 'reading', the 'normal', is retained in the information system, but this assumption must now be discarded. The various smoothing operations and regression operations which have been described do indeed result in a single output; however this figure is simply the central, most likely trend. Other values could be placed upon the variable being measured, which although they are less probable, still remain possible. At the outset it should be appreciated that these central trends which form the normal costs, etc. differ somewhat from the 'means' which are normally discussed in connection with 'probability'. Supposing weights of a series of identical small castings coming off a machine were

	ozs
(1)	2·3
(2)	2·1
(3)	2·4
(4)	2·2
(5)	2·5

The mean weight is the arithmetic average of all the observations, namely 2·3 ozs, the variance (v) is 0·02 oz, and the standard deviation (σ) is 0·139 oz.[2] If it is appropriate to make certain assumptions about the distribution of the actual weights about this mean, the variance and the standard deviation provide some measure of the probability that the mean will be equalled on any one occasion.

This type of calculation is suitable for use where a series of identical components are being made by the same machine over a period of time, which is short enough for the technological environment to be assumed to be stable. However this is not the case with most of the normal costs being considered here: in general a changing environment and technology are assumed, which typically is reflected in the exponential smoothing operation.

The smoothed normal is a weighted average, which is usually of considerable complexity, but it is probably not beyond arithmetic ingenuity to calculate variances for complex means of this type. It should be observed that only basic statements could possibly have variances in their own right; complex statements can only inherit the probabilities of their component basic statements. Before considering the 'calculus of probability' which would be necessary to compute the variance of a complex statement, or how the Lieberman matrix might be adapted to assist in this, it is necessary to enquire whether all basic statements may be expected to possess probabilities of this sort and whether this measure of probability truly represents every aspect of uncertainty which a manager would wish to consider in the course of business decision making.

6.3 PROBABILITY AND RELIABILITY

Most texts on probability theory distinguish carefully between statistical probability, which implies that the experiments resulting in the measurements can be repeated indefinitely, and Bayesian probability, which is conceived in terms of the decision maker's own prior confidence in some outcome which is modified in the light of additional experiments. Much of what has been said in this chapter and what has been implied in previous chapters can be seen to confuse the two concepts; unfortunately this seems unescapable in practical affairs! To the extent that it is possible to construct a series of Bayesian probability assessments, it may be possible to arrange these into a probability distribution, whose significance may differ from that for a statistical probability, but whose arithmetic properties might be similar (this technique is used commonly in 'sensitivity analysis' work, e.g. (51)).

Practical experiments in the construction of Bayesian distributions have not proved very successful; a man states, say, '£100', to which the decision maker counters, 'Would you say you were 90 per cent confident of that figure?' Whatever the respondent replies, it is obvious that in the end he will be able to say to his interrogator, 'Don't blame me, you suggested the distribution, I just said "£100"!'[3] It is possible that a more fruitful line of enquiry might be to look into the processes by which the respondent arrived at his initial reply of £100. He might have arrived at the amount by a flash of inspiration, in which case the amount of £100 must stand or fall on the confidence one felt in the statement; there would be no justification for inviting the respondent to supply artificial odds in such a case, since his 'inspiration' has supplied just one figure and has said nothing at all about any others. One might entertain a degree of confidence in the reliability of his inspiration, so that one might say, 'I think that there is a 90 per cent chance that he is right,' but it is hard to see in what circumstances one could, with reason, start assigning degrees of probability to different ranges of error about an assessment which genuinely was arrived at in this fashion.[4]

Winkler (116) examines another aspect of this problem in some detail. He points out that in theory, subjective probability distributions can be reduced to coherent probability measures, if the decision maker really knows something about the working of the phenomenon which is the subject of the decision. (It will be seen that if this knowledge of the relevant parameters is present, it may be possible to arrive at an *a priori* distribution on a more objective basis.) However if the decision maker has no such knowledge he will consult someone who has. He will insert authoritative statements about probability distributions into his data bank. Winkler then considers what would happen if one consulted more than one expert and so obtained a range of opinions on a single distribution. It seems reasonable to expect this range of opinions to form the basis of a single probability distribution and Winkler calls this 'the Consensus problem'. One solution would be to employ some weighing system to obtain an average distribution; another would be to consider each expert judgement as a further sample, which under certain circumstances could be incorporated into the currently held consensus in a manner similar to successive applications of Bayes' Theorem.[5]

Returning to the illustrations however, it will be found in practice that the estimate of £100 is not likely to have been snatched out of thin air. Instead any forecast figure will usually have been calculated as some sort of extrapolation of past experience; following the analysis developed in the previous two chapters, this indicates that the latter type of estimate must be a complex statement, while the former inspirational guess must be a basic statement and in particular an authoritative statement (which might also have an authoritative distribution supplied by the 'expert'). It follows that a Lieberman chart and matrix can be constructed for the calculated forecast, and the probability and reliability of the result will depend entirely upon the probability and reliability of the basic statements from which it has been constructed.[6] It seems useful to distinguish 'probability' from 'reliability'; the term 'probability' will be used only to describe statistical probability measures which have been calculated on the basis of a number of observations or authoritative statements, or which have been received as an authority in its own right. The term 'reliability' will refer to the level of confidence which is held in the authenticity (or 'credibility') of a basic statement, whether as a measure of an observed phenomenon or as an authoritative assessment of some state of nature.[7]

An internally collected observation will have a high reliability, in the sense of one's confidence in the accuracy and integrity of those who have collected it, but its standard deviation about the resulting mean could be substantial.

On the other hand, an authoritative statement will have no statistical probability, unless the source of the statement also supplies this *ex cathedra*, but will usually be awarded a reliability which is something less than 100 per cent.[8] It is significant that considerations of 'reliability' should be seen as playing a substantial part in accounting work. It has been pointed out that it is only in the sphere of social science that scientific observations have to be taken by the subjects of the experiments! The probable degree of the bias referred to in an earlier chapter will have to be taken into account in any attempt to measure this factor in respect of the enterprise's own employees, together with their level of skill and care in the performance of their duties. Outside 'authorities' would probably exhibit more complex reasons for bias, but both internal and external sources of data will be the subject of a feedback of the results of relying upon them in earlier periods.

The validity of the distinction drawn here between 'probability' and 'reliability' can be seen from the uses to which the two measures can be put. Both are necessary when the feedback of the results of the information system comes to be analysed in the next chapter. The feedback data consist mainly of the budget and cost variances experienced in the system; consideration of the statistical probability of the normal cost will determine whether a given variance is statistically significant or whether it can be ignored as being explained by the statistical 'noise' of the system. On the other hand, once a significant variance has been admitted, it is necessary to consider what action should follow, and how extreme that action should be. It seems reasonable to suggest that the strength of this reaction to a variance will be dictated by the confidence one has in the reliability of the sources of the data which produce it.

6.4 THE RELIABILITY OF COMPLEX STATEMENTS

Reliability is seen therefore as proceeding from some subjective confidence in the source of the data; a 98 per cent reliability implies a belief that in 98 cases out of 100 the information supplied will be of a satisfactory authenticity. It might be argued that this type of assessment is entirely idiosyncratic; A's reliance upon a given proposition is likely to be different from that of B. Hence the predominance of any one set of these assessments in an information system is likely to proceed from its holder's rank or personality rather than from any logical synthesis. It might seem that in such a case it would be impossible for anyone to 'audit' these assessments, other than by means of hindsight when the accuracy of the data has been demonstrated. The problem is not unique; statutory auditors have to certify the rationality as well as the arithmetic accuracy of a company's accounts. Thus in addition to verifying the cost of an asset, they also have to assess the reliability of the management's stated belief that the item will not become obsolete before the end of a certain 'life'. In this example, it would be very difficult to deny the validity of such a statement on a single occasion, say on the first audit of a new enterprise, but one would expect these assessments to remain consistent over a period of time. If the expected remaining life of a piece of plant at the end of 19x1 is five years, but this is increased to ten years at the end of 19x2, one could demand a detailed explanation of the reasons for the change. Similarly one might expect a certain consistency in the evaluation of sources of data over a period of time; moreover any changes in the assessments should be justified by reference to past experience.[9]

Especially since the data in the information system is commonly collected as a by-product of the housekeeping and control procedures of the real system, it is quite probable that more than one source might exist for the same data. In many cases these would be simply the material for some averaging operation such as has been described. However other situations might arise where certain sets of data were required to show a degree of consistency, and so confirm or deny the accuracy of other sources. If one source with a 98 per cent reliability reads "$X = £100$' and three sources with a 33 per cent reliability each say '$Y = £90$', where some logical reason exists for believing that $X = Y$, it is necessary to ask what figures should be taken up into the information system, for both X and Y, and further, what measure of reliability can be attached to them. It is suggested that in this case the figure of £100 would be accepted, and it would continue to carry a 98 per cent reliability. If another 98 per cent reliable source was added which read '$Y = £110$', one might accept an average of £105 and still retain 98 per cent reliability. The rule is therefore that information from equally reliable sources is to be averaged, and the average reading of the most reliable source(s) is to be adopted without altering its reliability with regard to contrary statements from less reliable sources, or inconsistent readings between sources of the same reliability. It is suggested that the reliability of the best source cannot be added to or reduced by readings from less reliable sources, at least while the *a priori* assessments of the reliability of the sources remain unchanged.[10]

If it is assumed that satisfactory measures of reliability can be attributed to the sources of the various basic statements, it will be a relatively simple matter to construct a calculus of reliabilities which can operate through the Lieberman matrix. Presumably it is always reasonable to assume in addition that all the basic statements must be independent variables; if one is prepared to take cognisance of the fact that the value of one variable depends upon the value of another, the former must be a complex statement, by definition. However the strong assumptions made in Chapter 5 in order to construct the semi-static model should not be forgotten in this connection. Returning to the lower-order complex statement whose construction was illustrated in Figures 5.4 and 5.5, these reliabilities might be attached to the basic statements therein:

		%	
(1)	Normal rack cost ^{t}	95	
(2)	Observed rack cost ^{t}	98	
(3)	Normal total cost ^{t}	90	
(4)	ψ^{t+1}	80	
(5)	Estimated total revenue $^{t+1}$	85	
(6)	$\sum$ Smoothed rack cost $^{t+1}$	94 [11]	
(7)	$\sum$ Smoothed organisation slack $+ \psi^{t+1}$	88 [11]	
(8)	Actual activity ^{t}	96	
(9)	E_i^{t+1}	90	
(10)	Normal organisational slack $^{t+1}$	100	(to be evaluated through η)

A further complication needs to be taken into account here; the reliability of the coefficients in the Lieberman chart itself may be less than 100 per cent. In general where the coefficient is shown as '1' or '×' or '÷', this indicates an arithmetic operation which is entirely neutral in the matter of reliability. However this can hardly be the case for the scaling coefficients, which might be given the ratings:

α	95%
β	95%
δ	80%

Some conceptual difficulty may be experienced here, since these items are respectively the weights used for exponential smoothing of the data, and the forecast-data multiplier. These are the subject of a feedback on the success of using them in forecasting work in the past, and so must be supposed to represent the best understanding of what the weights, etc. should be at this time. Nevertheless the mere fact that a piece of data cannot be improved upon does not mean that it must be awarded 100 per cent credibility. 'Reliability' is always a measure of confidence in a statement and does not necessarily reflect the good faith of the source. It might be noted that α and β are interdependent variables (since $\alpha + \beta = 1$), and they should therefore bear the same reliability rating. Where independent variables are multiplied together, so that

$$x_1 x_2 = z,$$

it seems appropriate to apply the normal Law of Multiplication from Probability Theory to the reliability of the product z (ρ_z). Thus

$$\rho_z = \rho_{x_1} \rho_{x_2}$$

Since α and β have only 95 per cent chances of reasonable accuracy, while δ has only an 80 per cent chance, it would seem that

$$\rho_{(\alpha)(\delta)} = \rho_{(\beta)(\delta)} = 76\%$$

On the other hand, when independent variables are added together, so that

$$x_1 + x_2 + x_3 + \ldots + x_n = z$$

the reliability of the total, z, is a weighted average of the reliabilities of $x_1, x_2, x_3, \ldots, x_n$:

$$\rho_z = \frac{\sum_i^n x_i \rho_i}{z}$$

This suggests that the reliability of any complex statement can be calculated from a slightly amended version of the Lieberman chart shown at the end of Chapter 5. The top righthand quadrant of the inverted matrix for statements 1 to 14 can be multiplied through by the appropriate reliability ratings to become

	(11)	(12)	(13)	(14)
(1)	(0·95 × 0·61) = 0·579	−0·95	(0·95 × −0·61) = −0·579	
(2)	(0·98 × 0·15) = 0·137			
(3)		0·90	(0·9 × 0·61) = 0·549	
(4)			0·80	
(5)				0·85
(6)				−0·94

(Notice how $\rho_{(\alpha)(\delta)}$ and $\rho_{(\beta)(\delta)}$ compound the initial reliability rating of the basic statements: i.e. '0·61' and '0·15' are respectively 0·80 × 0·76 and 0·20 × 0·76.)

These are the ρ_{x_i}'s which can be converted into $x_i\ \rho_{x_i}$'s and summed as follows:

	(11)	(12)	(13)	(14)
(1)	2·316	−3·80	−2·316	
(2)	0·685			
(3)		6·30	3·843	
(4)			0·240	
(5)				63,750
(6)				−47,000
$\sum_i^n x_i p i =$	3·001	2·50	1·767	16,750

The values of z in these cases were

	3·78	3·00	2·46	25,000

and therefore

$\rho_z =$	79%	83%	72%	67%

The calculation and the reliability of η can now continue, since

$\rho_{[15_1 x]} = \rho_{(8)} = 0{\cdot}96$

$\rho_{[15_2 \div]} = \rho_{(7)} = 0{\cdot}88$ (and $\rho_{\frac{(i)}{1}} \equiv \rho_{(i)}$)

so that

$\rho_{[15_3]} = \rho_{(13)} \times \rho_{(8)} \times \rho_{(7)} = 0{\cdot}72 \times 0{\cdot}96 \times 0{\cdot}88 = 0{\cdot}61$

and $\rho_{[15_4]} = \rho_{(9)} = 0{\cdot}90$

so that

$\rho_{[16]} = \rho_{(14)} \times \rho_{[15_3]} \times \rho_{[15_4]} = 0{\cdot}67 \times 0{\cdot}61 \times 0{\cdot}9 = 0{\cdot}37$

The calculation for column (17) of the Lieberman matrix ('normal cost per unit') is therefore

(1)		2·316
(2)		0·685
(10)	2·562 × 0·37 × 1·0	0·948
	$\sum_{i}^{n} x_i \, p_i =$	3·949

and $p_z = \dfrac{3{\cdot}949}{6{\cdot}342} = 0{\cdot}62$

It is apparent that the multiplication of only moderately unreliable basic data rapidly produces complex data which is very unreliable indeed. It may be that business-people would commonly express much greater confidence in their basic data than those ratings used in the illustration; this would be an interesting area for further research. (See also Note 12.)

6.5 THE NATURE OF OBSERVATIONS

It seems reasonable to look more closely into the operation described as 'observation'. In the latter part of this book, observed data have been treated as basic statements, to which single measures of reliability and probability are attached. However it might be possible to discern a number of steps in this operation which might complicate the analysis in various ways. An observation seems to involve (a) recognition, (b) allocation and (c) measurement.

(a) A piece of data is 'recognised' whenever the information system becomes aware that it has before it evidence of the existence of some transaction or state of nature, which is of a type which its policy (as described in Chapter 4) tells it is part of that subset of total information which is relevant, observable and commercially viable for collection. This is obviously a necessary prelude to any action by the information system in respect of the phenomenon; one might suppose that the system will be fairly reliable in its recognition of relevant, etc. data, but it is unlikely to be 100 per cent reliable, and its reliability will vary from enterprise to enterprise in this respect.

(b) Once recognised the piece of data needs to be recorded. In general it will be recorded in numerical form, but relevant information is not necessarily quantifiable, and in any case it will almost always carry with it a 'housekeeping tag' which is necessary to identify the data in the data bank, as opposed to measuring it. Thus it seems reasonable to say that an operation which allocates it to some category of data is needed before the phenomenon can be measured, or otherwise encoded into the information system. Any one piece of data can be allocated to a variety of categories, so that it is possible to talk of each transaction or state of nature possessing a number of aspects. The concept will be familiar to anyone who has worked with a 'logical' account-coding system whereby, say, 04/076/019 signifies 'Darlaston Works'/'Paintshop'/'Direct labour', while 06/076/112 signifies 'Durham Works'/'Paintshop'/'Consumable stores'. Here again the system might be expected to be more reliable in identifying some aspects than others, and less than totally reliable with any one of them.

(c) Finally the phenomenon is quantified, or otherwise encoded into a form which can be retained in the data bank, and if necessary be aggregated in some meaningful fashion with the other observations which have been allocated to the same category.

This makes it possible to extend the Lieberman chart (at least in theory) for any one 'observation' as shown in Figure 6.1 (assuming some reliabilities).

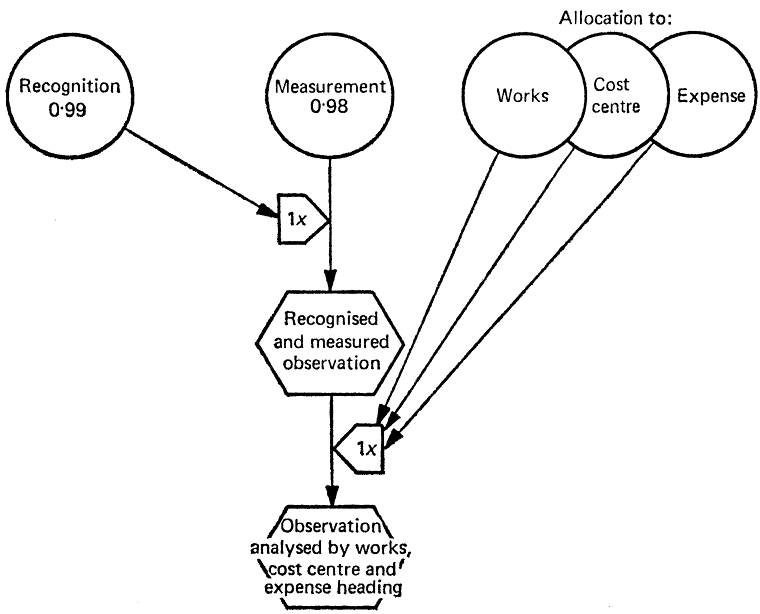
The effect of 'recognition' and the various 'allocations by aspect' is simply to multiply the basic 'measurement' by a scalar of unity, but a scalar *whose reliability is less than 100 per cent* the resulting calculations would show

Recognised and measured observation $= (0{\cdot}99 \times 0{\cdot}98) = 97\%$

Observation analysed by works, cost centre and expense heading $= (0{\cdot}97 \times 0{\cdot}98 \times 0{\cdot}96 \times 0{\cdot}97) = 88{\cdot}5\%$

The reason for treating 'recognition' separately from 'allocation' is that the data could be analysed by varying degrees of fineness. Thus an analysis by 'works' only would be 95 per cent reliable and by 'works and cost centres' 91·2 per cent reliable. It should be noted that these analysed totals are interdependent, so that if the 88·5 per cent reliable totals which have been analysed in full detail are then summed, the sum resulting will of course be 97 per cent reliable.[11]

FIGURE 6.1



6.6 THE PROBABILITY OF COMPLEX STATEMENTS

A similar procedure could be devised to provide a calculus of probabilities for use with the Lieberman matrix; the calculus of probabilities is of course better established (e.g. 49 and 74) than the calculus of reliabilities.[12] However before selecting some particular measure of probability, it is necessary to ask how the information about the probability of any basic statement is likely to be presented to the information system. Although we have rejected the application of Bayesian probabilities to complex statements, there is a possibility that much of the basic data will be presented to us in such terms as, 'I am 95 per cent confident that the figure is accurate within ± 20 per cent.' Some of course will be authoritative statements which have to be accepted as deterministic, but others will be in the form of replicated 'readings' which are suitable for the calculations of statistical means, standard deviations and so on. Bubb, Hussey and Smith (14) rightly emphasise the need to standardise one's measure in some way; if one likes to make some assumption about the nature of the distributions, it is possible to convert either of the measures used here into the other.

The example is continued as if the data had all been converted to 'basic standardised Bayesian' probabilities, which can be manipulated by the following *approximate* formulae for the calculus of probability:[13]

Addition $(A \pm a) + (B \pm b) = A + B \pm \sqrt{(a^2 + b^2)}$

Subtraction $(A \pm a) - (B \pm b) = A - B \pm \sqrt{(a^2 + b^2)}$

Multiplication $(A \pm a) \times (B \pm b) = A \times B \pm A \times B \times \sqrt{[(a^2/A^2) + (b^2/B^2)]}$

Division $(A \pm a) \,/\, (B \pm b) = A \,/\, B \pm (A/B) \times \sqrt{[(a^2/A^2) + (b^2/B^2)]}$

The confidence limits attaching to the basic statements are:

		Value £	95% Confidence limits Absolute	%
(1)	Normal rack cost ^{t}	4·00	± 0·40	± 10
(2)	Observed rack cost ^{t}	5·00	± 0·10	± 2
(3)	Normal total cost ^{t}	7·00	± 0·84	± 12
(4)	ψ^{t+1}	0·30	± 0	± 0
(5)	Estimated total revenue $^{t+1}$	75,000	± 15,000	± 20
(6)	smoothed rack cost $^{t+1}$	50,000	± 4,000	± 8
(7)	smoothed organisation slack +	20,000	± 3,000	± 15
(8)	Actual activity ^{t}	5,000	± 1,000	± 20
(9)	E_i^{t+1}	6,000	± 1,500	± 25
(10)	Normal organisational slack	1·00	± 0	± 0

Non-specified scalars like α, β and δ have ± 0 limits. The top righthand quadrant of the evaluated $[L]^{-1}$-matrix in Chapter 5 again provides the pattern for the necessary calculations (see page 171).

	(11) Value	(11) Error	(12) Value	(12) Error	(13) Value	(13) Error	(14) Value	(14) Error
(1)	$0\cdot72 \times \pounds4 = \pounds2\cdot88$	$\pounds2\cdot88 \times \sqrt{\frac{0\cdot4^2}{4^2}+0} = 0\cdot288$	$-1 \times \pounds4 = -\pounds4\cdot00$	$-\pounds4 \times \sqrt{\frac{0\cdot4^2}{4^2}+0} = -0\cdot4$	$-0\cdot72 \times \pounds4 = -\pounds2\cdot88$	$-\pounds2\cdot88 \times \sqrt{\frac{0\cdot4^2}{4^2}+0} = -0\cdot288$		
(2)	$0\cdot18 \times \pounds5 = \pounds0\cdot90$	$\pounds0\cdot90 \times \sqrt{\frac{0\cdot1^2}{5^2}+0} = 0\cdot018$						
(3)			$1 \times \pounds7 = \pounds7\cdot00$	$\pounds7 \times \sqrt{\frac{0\cdot84^2}{7^2}+0} = 0\cdot84$	$0\cdot72 \times \pounds7 = \pounds5\cdot04$	$\pounds5\cdot04 \times \sqrt{\frac{0\cdot84^2}{7^2}+0} = 0\cdot6048$		
(4)					$1 \times 0\cdot30 = 0\cdot30$	0 0		
(5)							$1 \times \pounds75{,}000 = \pounds75{,}000$	$\pounds75{,}000 \times \sqrt{\frac{15{,}000^2}{75{,}000^2}+0} = 15{,}000$
(6)							$-1 \times \pounds50{,}000 = -\pounds50{,}000$	$-\pounds50{,}000 \times \sqrt{\frac{4{,}000^2}{50{,}000^2}+0} = -4{,}000$
$\Sigma=$	$\pounds3\cdot78$	$\sqrt{0\cdot288^2+0\cdot08^2} = \pounds0\cdot288562$	$\pounds3\cdot00$	$\sqrt{0\cdot4^2+0\cdot84^2} = \pounds0\cdot930376$	$2\cdot46$	$\sqrt{0\cdot288^2+0\cdot6048^2+0} = 0\cdot669871$	$\pounds25{,}000$	$\sqrt{15{,}000+4{,}000} = 15{,}524\cdot2$

The remaining calculations are:

$$\varphi = \frac{(8) \times (13)}{(7)} = 0{\cdot}615$$

$$\text{and } \eta = \frac{(14) \times (\varphi)}{(9)} = 2{\cdot}562$$

Thus

$$(8) \times (13) = 5{,}000 \pm 1{,}000 \times £2{\cdot}46 \pm 0{\cdot}669871$$

$$= £12{,}300 \pm \left(£12{,}300 \times \sqrt{\frac{1{,}000^2}{5{,}000^2} + \frac{0{\cdot}6698\,71^2}{2{\cdot}46^2}} = \pm £4{,}155{\cdot}69\right)$$

$$\div (7) = \frac{£12{,}300 \pm £4{,}155{\cdot}69}{20{,}000 \pm \ 3{,}000}$$

$$= £0{\cdot}615 \pm \left(0{\cdot}615 \times \sqrt{\frac{4{,}155{\cdot}69^2}{12{,}300^2} + \frac{3{,}000^2}{20{,}000^2}} = \pm £0{\cdot}227342\right)$$

and

$$(14) \times (\varphi) = £25{,}000 \pm 15{,}524{\cdot}2 \times 0{\cdot}615 \pm 0{\cdot}22734^2$$

$$= £15{,}375 \pm \left(15{,}375 \times \sqrt{\frac{15{,}524{\cdot}2^2}{25{,}000^2} + \frac{0{\cdot}227342^2}{0{\cdot}615^2}} = \pm £11{,}111\right)$$

$$\div (9) = \frac{£15{,}375 \pm 11{,}111}{6{,}000 \pm \ 1{,}500}$$

$$= £2{\cdot}5625 \pm \left(£2{\cdot}5625 \times \sqrt{\frac{11{,}111^2}{15{,}375^2} + \frac{15{\cdot}00^2}{6{,}000^2}} = \pm £1{\cdot}95951\right)$$

Thus

$$(10) = £1 \pm 0 \times 2{\cdot}56 \pm 1{\cdot}95951$$

$$= £2{\cdot}56 \pm \left(£2{\cdot}56 \times \sqrt{\frac{1{\cdot}95951^2}{2{\cdot}56^2} + 0} = \pm £1{\cdot}95951\right)$$

So that (15) becomes the sum of:

(1)	£2·88	±0·288
(2)	£0·90	±0·018
(10)	£2·5625	±1·95951
	£6·34	$\pm\sqrt{2{\cdot}88^2 + 0{\cdot}018^2 + 1{\cdot}9591^2} = \pm £1{\cdot}98064$

It is necessary to enquire to what extent division or the calculation of small net balances from large sums is likely to enter into the normal work of the information system. Quite apart from the complexity of the calculations these operations are likely to introduce very large variances into the system. It is probable that they are rather common in practice, with the result that many practical examples of complex statements may have highly uncertain distributions.

It may be possible to make some further general statements about the effect of building complex statements out of basic data of varying degrees of reliability and probability, especially where multiplication and division occur.

(a) The multiplication of even slightly unreliable data rapidly produces considerable unreliability in the product.

(b) The effect of multiplying probabilistic data by decimal fractions such as α and β whose own error is nil, is a substantial relative increase in the statistical probability of the product.

(c) It must be remembered that the analysis in this chapter has continued to assume that the basic statements are independent, although many have been thrown up by the same system in previous periods. Moreover many of the probabilities would be very different in an *ex post* situation, so that a Markov chain approach would be appropriate: thus if $\pm\alpha$ *were* given an error factor of $\pm 0{\cdot}03$, but β in fact is $\beta - 0{\cdot}01$, α would become $\bar{\alpha} + 0{\cdot}01$ and have no error at all. Covariance is positive if the variables increase in unison and negative if large values in some are associated with small values elsewhere; mathematically positive covariance increases error and negative covariance reduces it. Intuitively one suspects that negative covariance will predominate in practical systems.

The combined effect is to increase the rigidity of the information system. Since normal accounting procedures seem to require the use of a good number of 'authoritative' statements (besides those statements mentioned in (b)), they may tend to produce complex statements with a fair degree of statistical probability, but comparatively low reliability. This may support the validity of what seems to be the common practice of decision makers, when it comes to 'feeding back' the actual results into the information system: very little allowance is made for statistical noise in the system, but comparatively little disturbance in expectation seems to be engendered by the resulting discrepancies. This aspect of the problem will be discussed further in the final chapter.

6.7 NOTES

1. For readers who prefer symbolic representation, one could designate

1. *Space of terminal acts:* $A = \{a\} \ldots$
2. *State space:* $\Theta = \{\theta\} \ldots$
3. *Family of experiments:* $E = \{e\} \ldots$
4. *Sample space:* $Z = \{z\} \ldots$
5. *Utility evaluation:* u (; ; ;) on $E \times Z \times A \times \Theta \ldots$
6. *Probability Assessment:* $P_{\theta z} \{\cdot \cdot |e\}$ on $\Theta \times Z \ldots$

and so write

a. The marginal measure $P'_\theta \{\cdot\}$ or $P\theta$ on the space Θ. We assume that P'_θ does not depend on e.
b. The conditional measure $P_z \{\cdot|e, \theta\}$ or $P_{z|e,\ \theta}$ on the sample space Z for given e and θ.
c. The marginal measure $P'_\theta \{\cdot|z\}$ or $P'_{\theta|z}$ on the space Z for given e but unspecified θ.
d. The conditional measure $P''_\theta \{\cdot|z\}$ or $P''_{\theta|z}$ on the space Θ for given e and z: the condition e is suppressed because the relevant aspects of e will be expressed as part of z.

The basic text for this type of approach is that by Raiffa and Schlaifer (91); its applications to information systems has been discussed by Marshall (77) and Feltham and Demski (37) (see Note 3 of Chapter 1).

2. $v = \dfrac{\sum_i^n (x_i - \bar{x})^2}{n}$ and $\sigma = \sqrt{v}$

hence $v = \dfrac{0^2 + 0{\cdot}2^2 + 0{\cdot}1^2 + 0{\cdot}1^2 + 0{\cdot}2^2}{5} = 0{\cdot}02.$

3. It might seem possible to argue that if a man expressed 100 per cent confidence that a figure might exceed, say £80, one could ask reasonably whether he would be prepared to trade in a right to a certain £80 for an x per cent chance of receiving £100, and so create a range of values and probabilities with some logical foundations. However this approach is not without difficulty since it is not true to say that any one person would equate a certain £81 with a 90 per cent certain £90. Other problems arise in interrogation of this sort. If the man agrees to rate £100 at 90 per cent and, say, £81 at 100 per cent, he will be tempted to make a possibly unjustified assumption of continuity if asked to assess £90.

4. Thus if we say 'We half believe you,' we do not imply some sort of probability distribution about a mean of £100. Instead we imply that our *reactions* to the receipt of the data will be less strong (half as strong?) than

they would have been had we had complete confidence in their source. This aspect of the matter will be explored further in Chapter 7.

5. Winkler's point is that a single 'authoritative statement' cannot form the basis of a probability distribution: however a series of statements from different authorities might be treated in this way. He does not consider the possibility that some 'experts' might be less authoritative than others, and a calculus of reliabilities will be outlined in this chapter to cover this. Of course once we start accepting the views of experts, they have largely made our decisions for us, and Mitroff's plea for a 'dialectical inquiring system' (79) urges a much less mechanical assessment of conflicting expertise than is implied by Winkler.

6. It might be argued that one could attribute some Bayesian level of confidence in a complex statement, quite independently of its constituent basic statements. The proposition is similar to one which has already been considered in connection with the modelling of the real system; it would be true where the information system (and hence, as has been shown, the underlying real system) is stable. When the systems are large, closely interrelated, and subject to rapid structural change, these appeals to past experience lose much of their validity.

7. This definition contrasts sharply with Ijiri's (63), which sees reliability in terms of the likelihood of success following the use of the data in decision making; that is, as inherent in the data rather than as something inherent in the *source* of the data. Clearly these opposing interpretations follow on from whether one believes in the possibility of constructing a general data bank or not. The thesis of this part of the book is that data are assembled (and partly processed) without specific regard to their eventual use(s). (See also Note 1 to Chapter 4 and Notes 2 and 3 of Chapter 1). Also, Ijiri's concept implies an unchanging world of constantly replicated decisions!

8. It is necessary to be certain of the distinction between an observation and an authoritative statement; in general this will not depend intrinsically upon whether a particular piece of data was gathered by an employee or an outsider, or even whether the figure was arrived at by guess or by measurement in the first place. Instead it seems to lie in the way in which the figure is accepted into the data system. Authoritative statements will be accepted *ex cathedra*, although not necessarily fully believed; however they will be accepted as the current state of this item until they are replaced by new 'authorities', and they will not be the subject of any smoothing operation. The term 'observation' implies that a number of similar observations are to be taken, which implies some smoothing or averaging is to occur. It can be seen that a singular measurement cannot have a standard deviation, and so is on all fours with a single guess, while a series of guesses about the same item will have a mean and hence a standard deviation. To return to the example of the man who made an estimate of £100; if in fact he did just make a guess, one could therefore say, 'He always overestimates, so I will take £90 as the mean.' Obviously, his statement is not being accepted *ex cathedra* at all, and the £90 is in fact a complex statement comprising his current guess, his previous guesses, and the relative actual results.

9. A similar argument can be used to defend the concept of 'economic

accounting' itself against critics who feel that estimates of the future cannot be made satisfactory bases for profit calculations. It would be the continual re-issue of *consistent* estimates which would provide the necessary support for the employment of the technique in practice. It might be possible to adduce a 'theory of subjective consistency' to provide a satisfactory intellectual basis for some degree of confidence in all types of subjective data. It is probably nonsense to claim that the acceptance of a consensus is true objectivity, if one means the consensus of views of many people at one point in time only. However if the consensus can be given a dimension in time also, it becomes possible to justify purely subjective views by reference to the changes in those views necessitated by experience during the intervening period.

10. It may be that Dr Mitroff would disapprove of this rule (see Note 5)! A possible modification which might be entertained is that a cut-off point might exist above which we would insist upon a full investigation of discrepancies between highly reliable sources.

11. It is reasonable to permit the reliability of the summations for all cost centres' data to differ from that arrived at by calculation of the same data for this particular centre; it is unlikely on the face of it that all cost centres' results should enjoy an equal reliability.

12. A certain amount of material 'along these lines' has appeared in the accounting journals. Much of it has followed a fairly straightforward application of probability to forecasting by Jaedicke and Robichek (66): a recent commentary has been given by Ferrar, Hayya and Nachman (38). Something closer to the approach used in this book is taken by Bubb, Hussey and Smith (14). Their paper starts from the assumptions (a) that it *is* possible to make sensible Bayesian assessments of the probability of complex statements of a *fairly* high order and (b) that the relative errors should be quite small (as well as independent). This book has taken a contrary view of (a), but (b) is interesting, especially since the authors claim some experience to back up the various limits suggested in their paper. The argument seems similar to that used in Chapter 3 to defend heuristic programming: the probability that a 'real-life' situation may be less susceptible to error than a made-up example. On the whole the assertion seems less convincing in this case, but it might be observed that Bubb, Hussey and Smith are prepared to include 95 per cent confidence limits of up to ± 30 per cent within the terms of this assumption. (The author's view is that it may be negative covariance which stabilises real-life examples rather than small error-factors.)

13. The author's suspicion is that it might be better to do it this way, not only because it enables one to use these simple formulae (in place of the more complex versions used in his thesis), but also because it accords with the arguments at the beginning of this chapter on the unsoundness of converting *ad hoc* Bayesian assessments into pseudo-statistical probabilities! A friendly 'expert' assures the author that this method of approximating 'the propagation of errors' would be considered rather rough-and-ready by, say, physicists, who do a lot of this type of calculation. He commends (and the author transmits) a small student text entitled *Errors of Observation and their*

Treatment by James Topping (1955), London. On the other hand, one should appreciate that *all* these methods involve some fairly strong assumptions about the shape of the probability distribution, and could be quite dramatically wrong if used in particularly unfortunate circumstances.

7 Feedback and the Information System

7.1 THE FEEDBACK OF VARIANCES

A substantial body of theory exists on the mathematical aspects of 'feedback', whereby a degree of automatic control can be applied to various engineering devices, so that they continue to home in on certain objectives, by reference to their current degree of deviation therefrom. In addition a number of attempts have been made to extend the use of these concepts to economic affairs, for example by Simon (102) and Tustin (109); however these latter adaptations have probably proved less useful than might have at one time seemed likely, for a number of reasons. Principally this may have been because engineering feedback is almost always 'continuous', which means that the phenomenon is being monitored all the time, so that corrective actions and reactions follow very closely upon one another. In economic matters, on the other hand, the feedback is almost always 'discrete', where an interval of months and years exists between the time when a corrective action is possible and that when its effect can be judged. Again some work has been done on engineering aspects of discrete feedback, but this may not apply very directly to problems of economic management, since, as has been demonstrated, the data which are being fed back are essentially unreliable. Moreover mechanical and electronic feedback always assumes that the people who are designing the system know what it is that they want the device to monitor; usually this will be 'stability' in some predetermined respect. Examples would be the temperature of a boiler, the tension on a spool, the speed of a motor.

In Chapter 4 it has been argued that the objectives of economic endeavour are much less clearly defined, and in any case the causal relationships of economic phenomenon are less certain for the most part. As a result corrective action is seen as requiring the exercise of 'commonsense', 'judgement' and 'experience', rather than an automatic reaction to a divergence from plan.

7.2 THE ANALYSIS OF VARIANCES

The whole tenor of this book has been a questioning of the real value of judgement and experience in the management of interrelated and rapidly changing systems. The justification for the latter part of the book is the additional light which an analysis of the probability and reliability of the data might provide on the problem of controlling the real system under such circumstances. There are two general classes of variances[1] which can be used to provide a feedback to the information system: budget variances between budgeted costs and standard costs,[2] and the cost variances between standard cost and actual cost. If one assumes that there is no discrepancy between scheduled production and actual production, the budget variances reflect the extent to which the selling and production scheduling operation have diverged from the budgeted sales and output.

As has been mentioned in Chapter 3, any understanding of the operative capacity of the plant depends upon the expectation of the load which can be scheduled for it, rather than any maximum physical capacity. Accordingly, the appearance of a budget variance could indicate (i) a need to amend the normal for the plant capacity if no idle time is present. However production scheduling is required to take account of the inventory and order book position so that this variance could also mean (ii) that the sales budget was incorrect (if there is a sales variance), or even (iii) that shortages of materials and components or breakdown time had necessarily distorted schedules. Alternatively if such a variance is also associated with an adverse usage or efficiency variance (so that the production scheduler is seen to be getting the appropriate 'hours' out of the plant but without achieving the required 'output'), this could indicate (iv) a need for the revision of the normal usages. Finally (v) it might simply mean that the production scheduling was inadequate. Similarly the emergence of an adverse cost-variance could signify either slack supervision and workmanship, or a need to revise the normal costs because they have proved to be unattainable. A favourable variance could indicate a need to tighten the standards of the normal costs, or overzealous supervision, to the extent that it is accompanied by other undesirable side effects. In short both budget variances and cost variances can be attributed to human failure or technological necessity, with the additional possibility that a budget variance might be the best response to some change of circumstance.[3]

Before examining the possibility of deciding which alternative reaction might be appropriate to a given circumstance, or the extent of the adjustment needed to give effect to it, it is necessary to decide how the variance can be measured. It is suggested that the division of total cost into a rack cost and a slack element enables this to be done in a more meaningful fashion. Some part of the variance may be attributable to purely technological factors; one example would be a mix-variance, where the non-standard mix was dictated by deficiencies in the supply of raw materials, and similar inescapable departures from the normal usages and efficiencies which are in effect changes in the rack cost of the operation. Other cost variances, whether favourable or adverse, reflect a failure by the workers and supervisors to keep to the 'bargain' struck during the budget-building negotiation. Since it has been seen that this slack is necessary to some extent, to secure the sound functioning of the real system, it is apparent that variances in this area are not necessarily only those which need to be corrected by increased attention to industrial discipline.

However this slack variance is in the nature of a residual figure, and as such it can be seen as the final recipient of the statistical noise in the information system. So far the analysis has been of the following type:

(a) *Calculation of cost variance*

	£'000	
Actual cost	1,900	
Normal cost	1,800	
'crude' variance	100	adverse
less variance in rack cost	20	adverse
'slack' variance	80	adverse[4]

(b) *Calculation of budget variance*

	£'000	
Budgeted cost of planned operations	1,850	
Normal cost of actual operations	1,800	
'crude' variances	50	favourable
less 'explicable' variances under headings (i), (ii), (iii) and (iv)	40	favourable
Budget variance attributable to heading (v)	10	favourable[4]

It is now necessary to ask whether these variances have any statistical significance, in view of what is known about the probability distributions associated with the complex statements used to calculate them. It may well be that the statistically significant variance may be less than the figure arrived at initially, for the reason that some part of any variance resulting from the comparison of probabilistic data could be explained as lying within the deviation which might have been anticipated to occur, in view of the dispersal of the actual observations of the phenomenon about the expected mean. The problem is usually discussed, if at all, in the literature of accounting as one of devising criteria for deciding whether the operation of a centre is 'under control' or 'out of control'. This could be dealt with by examining the individual productive acts, and by considering how often the readings fall beyond various multiples of standard deviations about the expected mean. Clearly the appearance of a number of readings in the outer ranges over a short period of time is an indication of loss of control; this technique was described by Gaynor some years ago (48). However a paper by Amicucci (2) offers another approach to this problem which seems more helpful in assessing the element of noise in variances of the type being considered here. The net effect of statistical noise over a period of time will tend towards zero. This means that the permitted tolerance for the cumulative totals of variance become proportionately (although not necessarily absolutely) smaller over time. Production does not always proceed steadily throughout the budget period; as a result the tolerance permitted to the interim cumulative totals of cost and consumption will need to take account of the proportion of the budgeted task fulfilled by the centre.

Here is the previous example broken down over four sub-periods:

(a)

	1	2	3	4	Total
	£'000	£'000	£'000	£'000	£'000
Actual cost	300	350	900	350	1,900
Normal cost	300	300	800	400	1,800
'Crude' variance	—	50A	100A	50F	100A
Less variance in rack cost	5A	10A	8F	13A	20A
Residual cost variance	5F	40A	108A	63F	80A

(b)

	1	2	3	4	Total
	£'000	£'000	£'000	£'000	£'000
Budgeted cost	400	400	650	400	1,850
Normal cost	300	300	800	400	1,800
'Crude' variance	100F	100F	150A	—	50F
Less explicable variance	10F	15A	25F	20F	40F
Residual budget variance	90F	115F	175A	20A	10F

Some of this data can be represented in a cumulative form:

	1	2	3	4
	£'000	£'000	£'000	£'000
(a) Budget cost	400	800	1,450	1,850
(b) Normal cost	300	600	1,400	1,800
(c) Actual cost	300	650	1,550	1,900
(d) Residual cost variance	5F	35A	143A	80A
(e) Residual budget variance	90F	205F	30F	10F

The residual variances (d) and (e) can now be calculated as percentages of the cumulative normal cost (b):

	1	2	3	4
	%	%	%	%
Residual cost variance	− 1	+ 6	+10	+5
Residual budget variance	−22	−33	+ 2	−0·5

Because statistical noise should cancel itself over time, it should be possible to set bounds at the various points in time, within which the apparent residual variances could be expected to be entirely due to this cause. Amicucci, who had been using a similar system for the control of laboratory costs, suggested that the limits could be established on the basis of experience. He saw it as something analogous to a cost reduction standard: '[an example] displays another possible management approach, which calls for an initial variance (January) of six per cent, but requiring this to be reduced to one per cent for the last quarter of the year' (op. cit. p. 12). It is apparent that Amicucci himself was unconcerned with any analysis of *why* the acceptable level of variance might be expected to be reduced over time. The probability that a reported deviation might be as large or larger than the expected deviation is considered by Bierman, Fouraker and Jaedicke (4); however they look at the problem from the point of view of flexible budgeting only. This means that they take the output level for any one period and calculate the probability that the reported variance could have a purely random explanation but do not consider that random disturbances of this type should cancel themselves out over time.[5]

If the period in the illustration is sufficiently long for the effect of noise to approach zero, and the errors have a normal distribution, the limit of tolerance at any point within the period could be stated in terms of standard deviations about the normal cost. Especially where output is planned over a period of time, a manager would want to know two items for control purposes:

(a) whether any overall variance *at all* is to be expected at the end of the period, and
(b) whether it is possible to assess at any point within the period that the process is, or is not, under control; this approach implies that the activity for the period under review is a multiple of observations which are being added to the n observations from which σ and σ^2 are being calculated.[6]

Both approaches imply that the technology is supposed to be static over the period of the observation(s), but this is in any case the assumption underlying the medium-range plan.

If the multiple-observations assumption is accepted, it seems reasonable to assess the results to date for the sub-periods on the further assumption that at the end of the four sub-periods in this illustration, respectively 25 per cent, 50 per cent, 75 per cent and 100 per cent of the observations should have been made. If there was any information which suggested the contrary, the percentages could be adjusted to accord with it.

Using A. Hald's *Statistical Tables and Formulae* (Wiley 1952) Table II ('The Cumulative Normal Distribution Function'), it appears that 12½ per cent of the observations in a normal distribution may be expected to fall below $-1 \cdot 15\sigma$; 25 per cent below $-0 \cdot 67\sigma$ and 37·5 per cent below $-0 \cdot 32\sigma$. This suggests that the limits of tolerance for the cumulative variance at the end of the first three sub-periods might be set at

(1) $\pm 1 \cdot 15\sigma$ (2) $\pm 0 \cdot 67\sigma$ (3) $\pm 0 \cdot 32\sigma$

The reasoning behind this calculation is that if 25 per cent of the observations in the whole period might be expected to fall beyond $\pm 1 \cdot 15\sigma$ of the mean, any variance of less than this amount at the end of the first quarter might safely be ignored. If the standard deviation of this particular item was in fact ascertainable, and was discovered to be £70,000, the limits would become

(1) ±£80,500; (2) ±£46,900; (3) ±£22,400; (4) nil

Expressed as percentages of the cumulative normal cost, these tolerances become

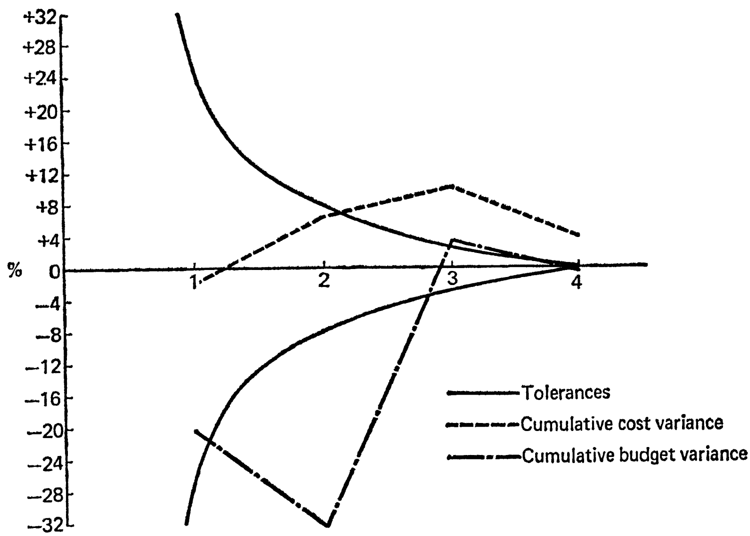
(1) ±26·8%; (2) ±7·8%; (3) ±1·6%; (4) nil

The data can be expressed in graph form, either in absolute terms, or as percentages (see Figure 7.1).

It can be seen from this chart that both costs and budgets were substantially 'out of control' in sub-periods 2 and 3; it is necessary to consider what must have occurred to bring about the improvement in the overall picture for the period. The variances which have been charted are of course the residual variances only, after the explicable or technologically necessary elements have been removed. The residuals may be assumed to be due either to shortcomings in industrial discipline or to incorrectly drawn standards which require amendment. These points will be discussed in detail in the following section; for the moment it will suffice to see that the improvement must be due either to improvements in discipline or to revision of standards.

Since it is being assumed that the standards themselves remain unaltered until the end of the period, presumably any revision of standards must have been included in the explicable or technologically necessary adjustments in subsequent periods.

FIGURE 7.1 Cost control chart



It seems reasonable to prefer this 'multi-observation' approach, since the basic assumption must be that the medium-range planning period is the period within which technological change is not to be expected. If this period does not contain enough observations for the distribution to settle down completely, the control lines will not reach zero at the end of the period, obviously; the controls might sometimes reach zero *before* the end of the period. It is assumed that cost investigations are always made, so that the explicable and necessary variances are eliminated before one considers whether the residual is under control. This can be contrasted with the treatment suggested by Bierman, Fouraker and Jaedicke (op. cit.), who see the main purpose of calculating the expectation of variance as leading to a decision as to whether or not there is need to investigate any given crude variance. This difference in the two methods is fundamental; Bierman, Fouraker and Jaedicke clearly see their technologies as stable, so that no substantial variances are expected. The method illustrated here on the contrary expects change and therefore expects variances which will require investigation. Also the philosophy expounded from Chapter 4 onwards implies that the standards themselves are imperfect, so that a continual feedback is needed to correct them.

Although the whole of the variance which lies beyond the tolerance needs to be fed back into the information system, it is also necessary to consider whether the variance lies in the rack costs or in the organisational slack. The strength of the required reaction to this feedback is governed by the reliability of the data from which it has been calculated. The mechanics of the feedback will be adjustments to the values of the scalars α, β and δ described in Chapter 5 for those variances discovered in the rack costs, and adjustments to ψ for variances which are attributable to the organisational slack.

7.3 THE RESPONSE OF THE INFORMATION SYSTEM TO FEEDBACK

Hofstedt and Kinard (60) have rightly drawn attention to the confused and confusing methodology employed by accountants when they attempt to explore the behavioural aspects of their subject; they are right too when they attribute this in part to similar deficiencies in the approach of professional psychologists to these matters! In discussing the budget negotiations, no attempt was made to elaborate upon ψ, the psychological element which disturbed the *status quo*. Unfortunately the response of the system to feedback can best be explained in behavioural terms, at least so long as the concepts of rack costs and organisational slack are accepted. Variances in the rack costs merely have to be evaluated as temporary or permanent disturbances in the technology of the enterprise, and either ignored except as part of the necessary variance described above, or adjusted into the rack cost. The residual cost variance can only be explained in terms of organisational slack.[7] In Chapter 4 this was seen as a response to the psychological needs of the supervisors and work force, and so it becomes necessary to interpret the response of the information system to feedback in this area, in terms of its understanding of the psychological attitudes of its employees.

The author might hope to disarm criticism from the more behaviourally sophisticated (see, for example, (8)) by suggesting the following 'Mickey Mouse' approach to this problem:[8]

(i) Assume that it is possible to evaluate in some way the psychological attitudes of the workers and supervisors,[9] and further, *to attribute these attitudes solely to their reaction to the current budget and its affect on their pay and conditions*. These attitudes can be arranged in some order of magnitude; if titles must be attached to them, they might range from 'extremely complacent' to 'mildly complacent', to 'mildly frustrated', and to 'extremely frustrated'.

(ii) Also assume that there is some optimal attitude[10] to be engendered in the workers, which lies within this range although not necessarily at its mid-point. Moreover the desirability of the attitude (from the point of view of the enterprise) is seen to fall away continuously from this optimum as the attitude moves towards the extremities of the scale.

(iii) If these assumptions are made it seems possible to suggest that where the residual variance is adverse, the appropriate channel for the feedback reaction would depend upon whether the rating attributed to the workers' attitude was above or below the optimal point on the scale:

(a) If the workers are more 'frustrated' when an adverse variance occurs (bearing in mind that it is assumed that this frustration is entirely attributable to the budget), this indicates that too hard a bargain has been driven with them at the previous negotiations and the standard should be revised. This reaction will take the form of a positive ψ; the *size* of the psychological disturbance of the *status quo* will be governed, in some fashion which will not be examined here, by the interaction of the size of the variance, the reliability of the complex statements which underlie it, and the amount of added value available for distribution.

(b) On the other hand, if the workers seem more 'complacent', the nature of the reaction to an adverse variance should be a tightening of industrial discipline, so that the negotiated bargains may be kept.

(c) Clearly it might be possible to have a favourable variance and nevertheless find 'frustrated' workers; if the optimal nature of the less frustrated attitude is agreed, one must suppose that the apparent financial benefit to the enterprise will be offset by greater militancy in future wage negotiations, although an even greater disadvantage might be to society as a whole, rather than to the enterprise itself. The appropriate reaction here might be a relaxation of discipline to enable the workers to achieve the organisational slack which it had been agreed that they should have.

(d) Finally complacency accompanied by a favourable variance would indicate that the bargain was too easy to achieve and the standards should be revised downwards.

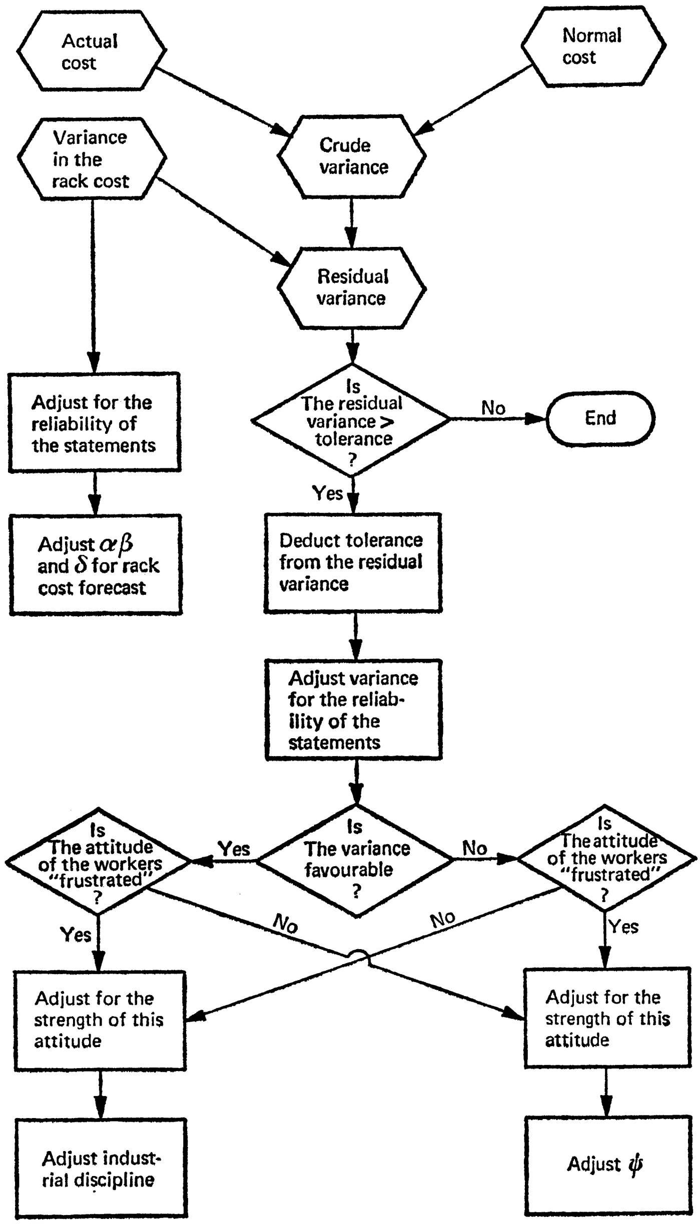
The 'Mickey Mouse' element of this analysis lies mainly in the assumption of the existence of an attitude to work which can be attributed solely to the budget situation; in fact this attitude will be inextricably concerned with many other facets of the workers' lives, both in the enterprise and in society at large. However the fact that such an assessment of attitudes must be largely subjective does not mean that it is completely worthless; here too, where the assessment is one of a series made over a long period, and amended in the light of experience of acting upon them, it gains a limited authenticity which is totally lacking in the single assessment itself.

It is submitted that the division of costs into 'rack cost' and organisational slack enables the accountant to speak with greater precision of 'controllable' and 'uncontrollable' cost-variances. By definition any variances in rack costs must be uncontrollable, in so far as they are technologically necessary. On the other hand, variances in the organisational slack must be controllable, at least in the sense that something can be done about them, either by varying the incidence of industrial discipline or varying the subsequent budget negotiation.

The mechanism of the feedback can be illustrated as a block diagram (Figure 7.2).

This scheme could be complicated by considering a number of subsidiary feedbacks. For example the 'adjustment for the strength of this attitude' must have its own strength governed by the success or failure of the subsequent corrective action in returning the employees' attitude to equilibrium. It is not suggested that these adjustments are the only influences which will be brought to bear on ψ or α, β and δ.

FIGURE 7.2 The feedback of variances



7.4 CONCLUSION: THE MEANING OF 'REALITY'

This book has been devoted to various aspects of the way in which the information system of an enterprise (basically its accounting system) reflects the underlying reality of its structure and operations. In its various chapters it has adopted two apparently contradictory theses; that it is unsafe to build any model of the enterprise upon anything but detailed analysis of its structure, but that any understanding of its technological structure, any understanding of its costs and profits and even the real behaviour of some of its costs are largely subjective. In fact this paradox is latent in the study and management of all economic affairs by man, since he is both the observer and the observed in these matters. This has led reasonably enough to a heavy reliance on experience and consensus of opinion in comprehending and regulating these matters. It is significant that some writers on accounting define 'objectivity' in terms such as the avoidance of *personal* bias, and the acceptance of a general consensus of views.

Unfortunately the rapid growth of the rate of technological change in recent times has meant both that there is insufficient time for this consensus to be formed and also that the situation might become too complex to be safely comprehended by pure intuition.

At the same time very little has happened to make these appreciations of economic affairs less subjective in themselves; the problem becomes that of comprehending and regulating a protean real system in largely subjective terms. This topic is one of considerable interest, which has been and is discussed by some of the most distinguished figures in the fields of economic analysis; consider this quotation from G. L. S. Shackle, *Decision, Order and Time in Human Affairs* (Cambridge 1969) 2nd ed.:

> Let us make a supposition about the nature of things, namely, that the rival possible outcomes which a man will imagine for an available act of his own cannot be listed from a knowledge, however complete, of what is and what has been. Two things amongst others follow. Decision, by which a man finds and adopts that one amongst his available acts which

promises or suggests the outcome that he most desires, is more than mere response to circumstances and contains an element which we may call inspiration, which brings essential novelty into the historical sequence of states of affairs. Decision thus becomes the locus of unending creation of history, and acquires the meaning which intuition and working attitude to life give it, in contrast to the character, implied for it by those who seek a sequential calculus of human conduct, of a passive link in chains of necessity. Secondly, in analysing decision, the use of a distributional uncertainty variable, that is, probability, becomes in principle inappropriate and must give way to a non-distributional uncertainty variable such as possibility, understood as discriminable in some manner into degrees; for example, by being identified with potential surprise. Such is the kernel of this book's argument . . . (preface to the first edition).

It may be that a further exposition of these ideas lies outside what even the author envisages as the proper study of accountants. It might be reasonable to argue that accountants are the engineers of the information system, so that anything proposed in a book dealing with accounting should look forward to the possibility of implementation. Even though the material in the last four chapters has not so far been applied in the field the author is at least committed to its eventual application in real-life industrial situations. Professor Shackle's proposition rather damages what many accountants would see as their main area of influence, since he states that possible outcomes and the degree of probability which attaches to them, cannot be ascertained purely from historical data, or from any extrapolation thereof which might be suggested by the previous history of events. Perhaps, just as the model of the information system presented in this book made only the vaguest suggestions about the composition of 'ψ', in deference to the psychologists, it may be that the model has been left equally open-ended in this respect also. Provision has been made for two types of basic statement to be picked up into the normal data; observations and authoritative statements. Something was said in Note 8 to the previous chapter of the need for care in distinction between the two. It may be that the distinction given therein can be made to conform with Professor Shackle's requirements; whether it has been created by 'guess' or by 'measurement', an 'observation' implies a multiplicity of similar statements and so leads on to the necessity of assigning some measure of probability. By contrast an 'authority' implies *ex cathedra* acceptance of a singular statement, and hence no meaningful probability can be assigned. However it was suggested that a further concept of 'reliability' could be introduced; this seems to have some affinity to the 'non-distributional uncertainty variable such as possibility' referred to in the quotation. On page 92 (op. cit.) the term possibility is contrasted with probability as follows:

> When a man speaks of probability he pushes ignorance as far as he can into concealment and an inconspicuous role; when he speaks of possibility he recognises his knowledge as merely setting bounds to a wide spectrum of mutually exclusive ideas. To sum the degrees of possibility assigned to various rival hypotheses is to fall back on the idea that it is the *number of its rivals* which gives a hypothesis its status, rather than its own particular character.

It would seem that the model proposed here continues to provide an adequate element of neutrality in respect of the decisions or even the philosophies of decision making which may be based upon it. In particular the essentially subjective nature of the normal data is recognised frankly, as is the indeterminate nature of the problems which may need to be resolved with their aid.

The present work proffers several techniques which may make this easier to achieve:

(i) The technological modelling technique, which at least assures that any subjective assessments of the economic situation are comprehensive, up to date and mutually consistent

(ii) The theory of economic accounting, as well as being necessary if the information system of the organisation is to correspond exactly with the currently-held model of the underlying reality, is also the method of calculating the income of an enterprise in terms which coincide best with economic analysis – and commonsense

(iii) The Lieberman model of the information system itself, which shows how the information system responds to a variety of external stimuli to form the currently held subjective beliefs about the structure, history and prospects of the organisation. (This also makes possible some analysis of the probability and reliability of the complex statements and also a study of the various feedbacks of information which are seen to play so substantial a part in providing a degree of support for data which of themselves are not directly verifiable.)

(iv) What might be called 'the behavioural theory of costs', which both explains some otherwise very puzzling problems in industrial accounting and provides a link between the technological aspects of the underlying reality and the psychological influences which are so inextricably intermingled with them. (In time, it may be possible to expand this material somewhat and give greater precision to the understanding of the apparent conflict between reality and its appreciation.)

7·5 NOTES

1. The term 'variance' is used here in its normal standard-cost accounting sense; this is of course quite unconnected with the statistical variance (σ^2) which was referred to in the previous chapter. The latter is a measure of the dispersal of the observations about the mean.

2. Of course, both Samuels (97) and Demski (27) would want to divide this variance into a true budget variance between the *ex ante* and the *ex post* (optimal) budgets, and an opportunity cost between the *ex post* budget and the standard costs. The trouble with this approach arises from something already discussed in Chapter 3; the mathematical connection between the L.P. solution and the daily schedules of production is not yet fully developed. This is because the pattern of orders and machine breakdowns *within* the period is not knowable in advance. Thus the 'opportunity cost', though 'correct', is not usable as a variance; one could not commend or take to task a production controller over its dimension!

3. An interesting possibility along these lines is discussed by Ronen (95) in a paper wherein he shows how idle capacity might be *planned for* so as to meet future demands. This is a good example of the tendency of conventional accounting to ignore all dynamic aspects of the system it is attempting to portray.

4. Note that both these residual variances are strictly 'controllable', to the extent that they are not to be accounted for as statistical noise.

5. A large part of their interest in this matter, as with some other writers, is over the decision to be taken as to whether or not to investigate a variance.

A more recent commentary has been supplied by Ozan and Dyckman (87).

6. This is another 'procedure' which should perhaps have been dealt with more fully in Chapters 4, 5 and 6. As well as maintaining and updating the normal data (presumably at discrete intervals) in the way described at some length, one would want to keep an eye on the historical data more or less continuously. There is no great virtue in waiting until '$t+1$' in order to press the panic button! This is why we might want to know the statistical variance of complex statements as well as the overall 'error': these can be calculated through the $[I-L]^{-1}$-matrix in much the same way.

7. The residual budget variance was attributed to ineffective scheduling (in Section 7.1), to the extent that machine interference and so on had already been allowed for; this analysis does not admit slack into the budget variances. That is not to say that the production controller has no slack to play with! It should be noted that the negotiation of the budget is just one of a series of negotiations which occur: the capacity of the plant and the 'ratio units' (see Chapter 5) are also the subject of negotiation rather than observation. Obviously the production controller is one of the major beneficiaries of the slack in these items.

8. It may be that very little apology is needed for such an approach. The author has discussed these ideas informally with a considerable number of senior managers, industrial accountants, supervisors and trades-union organisers; there seems to be pretty general agreement upon the truth of the earlier analysis of the nature of the budget negotiations. It is very hard to conceive of any laboratory experiment to demonstrate this truth, which was not open to the same criticism as that levelled against Stedry's work, namely that real decision makers in a real situation might exhibit different reactions to university students playing games with jugs of water! An empirical investigation of a real work-situation would seem to be very difficult to arrange.

From Westinghouse onwards very valuable work has been done on workers' attitudes to various aspects of their conditions, but the negotiations which trade off industrial discipline, productivity and reward for the workers, as a part of the general distribution of the added value accruing in the enterprise between the various grades of employees, the proprietors and internally financed growth, must be an unusually delicate area for research. It lies very close to the heart of what the whole system is about; it might have been equally difficult to do research on religious mania in Salem in the seventeenth century!

9. When we are discussing human reactions to feedback in this section it will be seen that the 'humans' are being considered as employees and supervisors – or even as 'collaborators' in the sense of the Behavioural Theory of the Firm. Thus their reactions to feedback as decision makers are not specifically discussed: a recent paper by Mock (80) deals with the latter topic. Since one's 'decisions' in real life have to be made as a member of a number of groups and are made on the basis of an interior mazeway rather than specific accounting statements themselves, it may be that this aspect of the problem is only marginally influenced by the accounting system.

10. The author is not being too specific as to what scale this 'optimality' is being measured in: readers may recall that what Becker and Green were taking Stedry to task over was an assertion that high morale produced high productivity! Perhaps if one talked of a 'morale/productivity ratio' one would allow for the situation where the best productivity was achieved at a level of frustration just short of insurrection (see Figure 7.3).

FIGURE 7.3

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